Spherical harmonics

laplacian ($\nabla\cdot\nabla$ or ∇^2) $= 0$

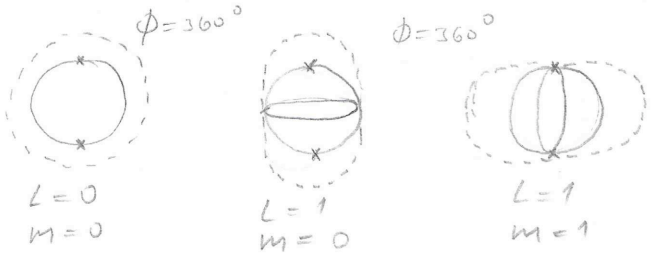
- homogenous polynomial solutions of laplace equation $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = 0$
- complex valued spherical harmonics are eigen functions of the square of the orbital angular momentum operator:

$$-i\hbar \vec{r} \times \nabla$$

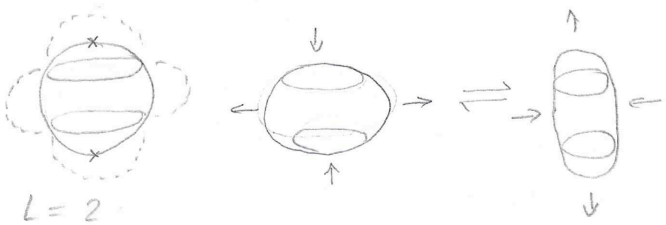
so solve schrödingers potential well and give solutions representing differently quantised configurations of atomic orbitals

- functions of an angle and can be +ve or -ve

L - number of nodal circles
m - number of nodal circles passing through poles so $L \geq m$



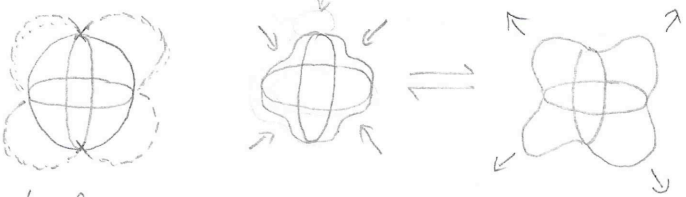
- surfaces on opposite sides of nodal circle oscillate in opposite directions so at a phase shift of π or $180°$

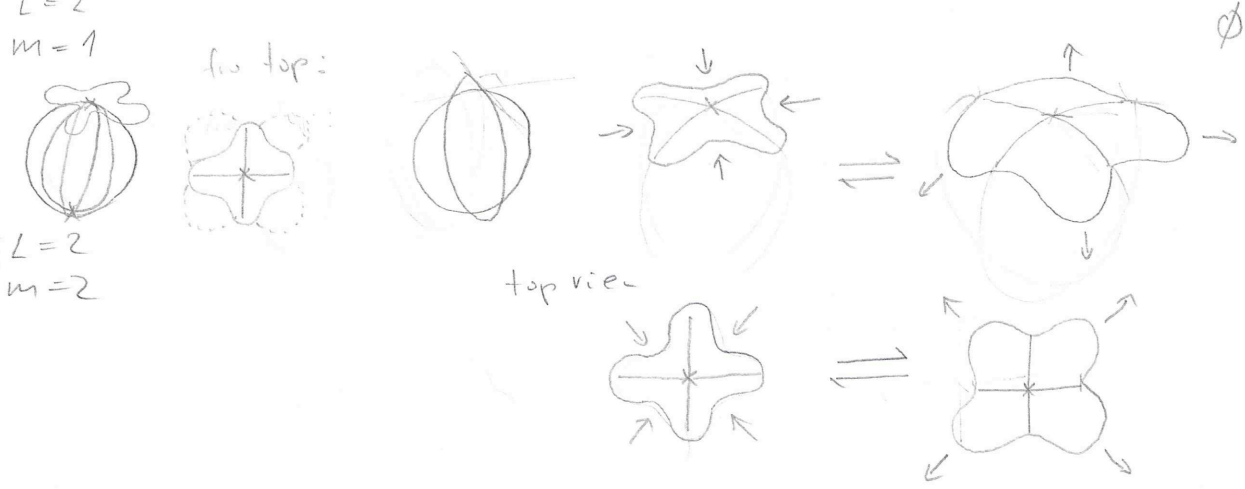


- ϕ azimuthal angle between nodal circles passing the pole

$$\phi = \frac{360}{2n}° \text{ or } \frac{2\pi}{m} \text{ rad}$$

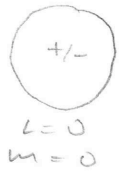
$\phi = 180°$ - divides polar circle into azimuthal angle ϕ



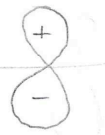


- nodal circles distributed symetrically between poles of the sphere

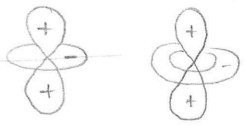
- on a parametric plot where distance r from centre at an angle θ is a point representing magnitude $u(r, \theta)$ of the spherical harmonic function

$L = 0$, $m = 0$: equal amplitude in all directions $\Rightarrow$ just a sphere $\rightarrow$ 

s

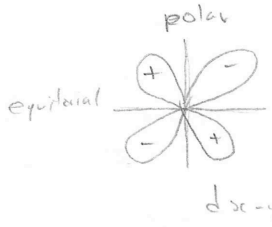
$L = 1$, $m = 0$: 1 nodal circle so 1 pinch and 2 loses of opposite direction equal magnitude since phase shift $\rightarrow$ 

p

$L = 2$, $m = 0$: 2 loses of equal sign gives 2 pinches since 2 nodal circles but they are both in 1 plane since both are equitorial - circle upsets region in between which is out of phase $> \pi$ but the two polar regions are in phase since 2 equitorial nodal circles $\rightarrow$ 

d

$L = 2$, $m = 1$: 4 loses on diagonals of opposite sign since 4 regions 2,2 out of phase



2 pinch lines since 1 represents equitorial and one represents polar nodal plane

- pinch represents a nodal circle and line of pinch represents weather nodal circle is equitorial or polar

angle between loses = ϕ ?

$$\phi_{mL}(\theta) = \frac{1}{\sqrt{2\pi}} e^{im\theta}$$