1

$I = \begin{pmatrix}1&0\\0&1\end{pmatrix}$ $\begin{pmatrix}a&b\\c&d\end{pmatrix}\begin{pmatrix}p&q\\r&s\end{pmatrix} = \begin{pmatrix}ap+br & aq+bs\\cp+dr & cq+ds\end{pmatrix}$ associative: $A+(B+C)=(A+B)+C$

$A(BC)=(AB)C$

$M=\begin{pmatrix}a&b\\c&d\end{pmatrix}$ maps $\begin{pmatrix}x\\y\end{pmatrix}$ $M\begin{pmatrix}x\\y\end{pmatrix}=\begin{pmatrix}x'\\y'\end{pmatrix}=\begin{pmatrix}ax+by\\cx+dy\end{pmatrix}$ commutative: $A+B=B+A$

$AB \neq BA$

- transformations:
  - ↳ reflection in x axis $=\begin{pmatrix}1&0\\0&-1\end{pmatrix}$ in y axis $\begin{pmatrix}-1&0\\0&1\end{pmatrix}$
  - ↳ reflection in $y=x$ $\begin{pmatrix}0&1\\1&0\end{pmatrix}$ in $y=-x$ $\begin{pmatrix}0&-1\\-1&0\end{pmatrix}$
  - ↳ rotation anticlockwise by θ about $(0,0)$ $\boxed{\begin{pmatrix}\cos\theta & -\sin\theta\\ \sin\theta & \cos\theta\end{pmatrix}}$
  - ↳ enlargement scale factor k about $(0,0)$ $\begin{pmatrix}k&0\\0&k\end{pmatrix}$
  - ↳ stretch parallel to x axis scale factor k $\begin{pmatrix}k&0\\0&1\end{pmatrix}$
  - ↳ stretch parallel to y axis scale factor k $\begin{pmatrix}1&0\\0&k\end{pmatrix}$
  - ↳ shear x axis constant with $(0,1)$ mapped to $(k,1)$ $\begin{pmatrix}1&k\\0&1\end{pmatrix}$
  - ↳ shear y axis constant with $(1,0)$ mapped to $(1,k)$ $\begin{pmatrix}1&0\\k&1\end{pmatrix}$
- transformation M followed by N is NM

for invariant point: $M\begin{pmatrix}x\\y\end{pmatrix}=\begin{pmatrix}x\\y\end{pmatrix}$

for invariant line: $\begin{pmatrix}x'\\y'\end{pmatrix}=M\begin{pmatrix}x\\y\end{pmatrix}$ $y=mx+c$, $y'=mx'+c$

$\det M = \frac{A'}{A}$

$M^{-1}=\frac{1}{\det M}\begin{pmatrix}d&-b\\-c&a\end{pmatrix}$ $M=\begin{pmatrix}a_1&b_1&c_1\\a_2&b_2&c_2\\a_3&b_3&c_3\end{pmatrix}$ $\det M = |M| = a_1\begin{vmatrix}b_2&c_2\\b_3&c_3\end{vmatrix} - b_1\begin{vmatrix}a_2&c_2\\a_3&c_3\end{vmatrix} + c_1\begin{vmatrix}a_2&b_2\\a_3&c_3\end{vmatrix}$

minor: $C_2 = \begin{vmatrix}a_1&b_1&\cancel{c_1}\\ \cancel{a_2}&\cancel{b_2}&\cancel{c_2}\\a_3&b_3&\cancel{c_3}\end{vmatrix} = \begin{vmatrix}a_1&b_1\\a_3&b_3\end{vmatrix}$ cofactor: C_2 is minor with the correct sign assiged by: $\begin{pmatrix}+&-&+\\-&+&-\\+&-&+\end{pmatrix}$

adjugate or adjoint $\operatorname{Adj}M=\begin{pmatrix}A_1&B_1&C_1\\A_2&B_2&C_2\\A_3&B_3&C_3\end{pmatrix}^T = \boxed{\begin{pmatrix}A_1&A_2&A_3\\B_1&B_2&B_3\\C_1&C_2&C_3\end{pmatrix}}$ singular if $\det M=0$

$M^{-1}=\frac{1}{\det M}\operatorname{adj}M$ $A^{-1}A=AA^{-1}=I$ singular if it maps all points into a lower dimension

plane: $ax+by+cz=d$

$\vec{n}=\begin{pmatrix}a\\b\\c\end{pmatrix}$

plane through: l_1 and l_2 $\vec{d_1}\times\vec{d_2}=\vec{n}$

OA, OB, OC $\boxed{\vec{r}=\vec{OA}+\lambda\vec{AB}+\mu\vec{AC}}$

plane through OA perpendicular to $\vec{n}$ $(\vec{r}-\vec{a})\cdot\vec{n}=0$

- cross product of $\vec{a}$ and $\vec{b}$ perpendicular to both $\vec{a}$ and $\vec{b}$

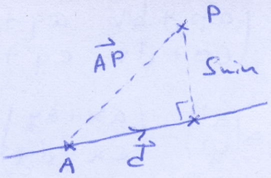
$\vec{a}\times\vec{b}=|\vec{a}||\vec{b}|\sin\theta\,\hat{n}$ $\vec{a}\times\vec{b}=\begin{pmatrix}a_1\\a_2\\a_3\end{pmatrix}\times\begin{pmatrix}b_1\\b_2\\b_3\end{pmatrix}=\begin{pmatrix}a_2b_3-a_3b_2\\a_3b_1-a_1b_3\\a_1b_3-a_2b_1\end{pmatrix}$ $\vec{a}\times\vec{b}=-\vec{b}\times\vec{a}$

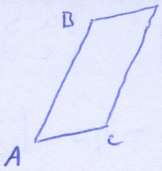
$(m\vec{a})\times(n\vec{b})=mn(\vec{a}\times\vec{b})$

$\vec{a}=\begin{pmatrix}a_1\\a_2\\a_3\end{pmatrix}$ $\vec{b}=\begin{pmatrix}b_1\\b_2\\b_3\end{pmatrix}$ $\vec{a}\times(\vec{b}+\vec{c})=\vec{a}\times\vec{b}+\vec{a}\times\vec{c}$

$\boxed{\text{if } \vec{a},\vec{b} \text{ parallel } \vec{a}\times\vec{b}=0}$

- distance
  - point to line: $\boxed{S_{min} = \left| \frac{|\vec{AP} \times \vec{d}|}{|\vec{d}|} \right|}$





Area $= |\vec{AB} \times \vec{AC}|$

  - two skew lines: $\vec{r_1}, \vec{r_2}$ $\boxed{S_{min} = \left| \frac{(\vec{d_1} \times \vec{d_2})}{|\vec{d_1} \times \vec{d_2}|} \cdot (\vec{a_1} - \vec{a_2}) \right|}$
  - point to plane: $\boxed{S_{min} = \frac{|a x_1 + b y_1 + c z_1 - d|}{\sqrt{a^2+b^2+c^2}}}$

angle between plane and line
$\cos\alpha = \frac{\vec{d_1} \cdot \vec{n_1}}{|\vec{d_1}||\vec{n_1}|}$
$\boxed{\theta = 90 - \alpha}$ (!)

  - parallel line to plane: choose a point on line then point to plane

- series is the sum of terms of a sequence $S_n = a_1 + a_2 + a_3 + \dots a_n = \sum_{r=1}^{n} a_r$

$\sum_{r=1}^{n} r = \frac{1}{2} n(n+1)$ $\quad \sum_{r=1}^{n} r^2 = \frac{1}{6} n(n+1)(2n+1)$ $\quad \sum_{r=1}^{n} r^3 = \frac{1}{4} n^2 (n+1)^2$

$\sum_{r=1}^{n} x^r = \frac{x(x^n - 1)}{x-1}$ (or use geometric seriese) method of differences - expand and then cancel out

- proof by induction
  - 1.) prove for $n=1$ 2.) induction hypothesis: assume true for $n=k$ then ...
    3.) show if true for $n=k$ then true for $n=k+1$ as well
    4.) we have shown if hypothesis is true for $n=k$ then it must be true for $n=k+1$ as well and since it is true for $n=1$ by the principle for mathematical induction it is true for all n, $n \in + \mathbb{Z}$

- quadratic - cubic - quartic $\sum\alpha = -\frac{b}{a}$ $\sum\alpha\beta = \frac{c}{a}$ $\sum\alpha\beta\gamma = -\frac{d}{a}$ $\sum\alpha\beta\gamma\delta = \frac{e}{a}$

$\boxed{\sum\alpha^2 = (\sum\alpha)^2 - 2\sum\alpha\beta}$ $\quad W = Z^2 \Rightarrow Z = \sqrt{W}$ or $W = Z+1 \Rightarrow Z = W-1$

- rational function can be expressed as $y = \frac{f(x)}{g(x)}$ $f(x), g(x)$ are polynomials $g(x) \neq 0$

- to sketch $y = f(x)$
  - find intercepts ($y=0$, $x=0$)
  - find asymptotes (vertical $y \to \pm\infty$ horizontal $x \to \pm\infty$ and oblique)
  - find stationary points

- to sketch $y = \frac{1}{f(x)}$
  - $y = f(x)$ and $y = \frac{1}{f(x)}$ intersect at $x = 1, -1$
  - $y = f(x)$ and $y = \frac{1}{f(x)}$ have the same sign
  - roots of $f(x)$ are asymptotes of $\frac{1}{f(x)}$
  - $f(x)$-increasing $\frac{1}{f(x)}$ - decreasing $\quad f(x)$ - minimum $\frac{1}{f(x)}$ - maximum

as $f(x) \to \pm\infty$ not shown as an asymptote but a discontinuity (open circle)

- sketch $y^2 = f(x)$
  - ↳ think of it as $y = \pm\sqrt{f(x)}$ - find $y = \sqrt{f(x)}$ and reflect over x axis
  - ↳ at -ve values of $f(x)$ undefined
  - ↳ if $f(x) > 1$ points pulled down if $0 < f(x) < 1$ points pulled up
    $y = 0$, $y = 1$ invariant so $y^2 = f(x)$ intersects $y = f(x)$
  - ↳ same vertical asymptotes horizontal asymptotes $y = k \rightarrow y = \pm\sqrt{k}$
  - ↳ type of turning point same but y coordinates $y \rightarrow \pm\sqrt{y}$
- converting polar to cartesean
  $r = \sqrt{x^2+y^2}$ $\theta = \tan^{-1}\left(\frac{y}{x}\right) \pm \pi$ if necessary $x = r\cos\theta$ $y = r\sin\theta$
- area of sector on polar graph $\boxed{A = \frac{1}{2}\int_{\alpha}^{\beta} r^2 \, d\theta}$