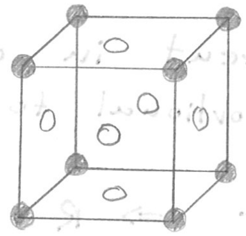
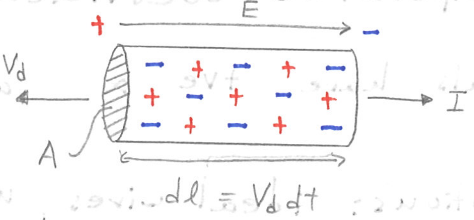
Basic Electronics 2021



- in solid copper Cu atoms are in a face-centered cubic arrangement
- each Cu atom has $29e^-$, $28e^-$ are loosely bound and $1e^-$ is free to dissociate

$$I = \frac{dq}{dt}$$

- conventional current - rate of flow of +ve charge $+ \rightarrow -$



- conduction speed

drift speed $V_d = \frac{dl}{dt}$ $\qquad V_d = \frac{I}{Ane}$

conduction e^- density $n = \frac{N}{V}$

$$I = nAV_dq$$

- origin of resistance
  - ↳ charge carriers (free e^-) experience constant acceleration due to the electric field
  - ↳ they undergo collisions with atoms causing them to slow down (don't move in straight line) and travel at average drift velocity

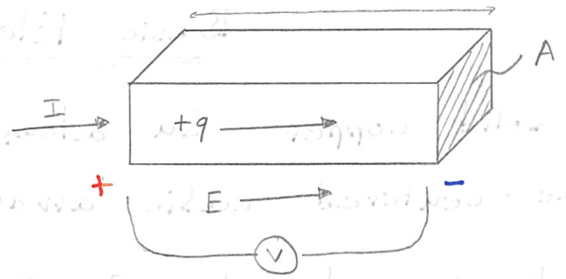
- thermal motion
  - ↳ conductor is not at 0K so random thermal motion of atoms which ↑ rate of collisions so ↑ resistance
  - ↳ when e^- collide with atoms it passes on some of its KE resulting in heating of the conductor

- electric potential
  - ↳ potential energy per unit charge $V = \frac{U}{q}$

- potential difference
  - ↳ change in electric potential $\Delta V = V_{ab} = V_b - V_a = \frac{U_b - U_a}{q}$

Ohm's law

↳ current in a conductor is proportional to the electric field



$$J = \frac{E}{\rho} \Rightarrow R = \frac{V}{I} = \frac{\rho l}{A}$$

resistivity and temperature $\rho = \rho_0(1 + \alpha(T - T_0))$

α - temperature coefficient ρ_0 - resistivity at T_0

↳ metals have +ve α and carbon has -ve α

assumptions: ideal wires, no loading, resistance fixed at rtp

electromotive force - EMF

↳ every C of charge gets $\mathcal{E}$ energy as it passes across source

↳ causes the charges to flow by generating E field

- charge in a circuit is allways conserved

- real voltage source

↳ finite capacity of energy $V = \mathcal{E} - Ir$

↳ internal resistance r

- ground

↳ electrical potential of our planet defined to be $V = 0$

- electrical power

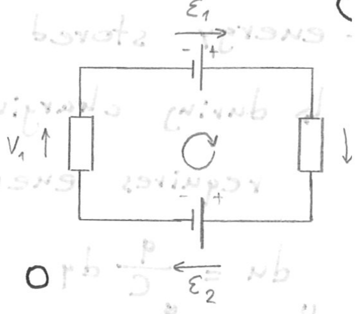
↳ E is conserved so E from source must be lost in resistor

$$P = \frac{dE}{dt} = \frac{dE}{dq}\frac{dq}{dt} = VI$$

↳ power in source: $P_s = -\mathcal{E}I = -\frac{\mathcal{E}^2}{R}$

power in resistor: $P_R = \mathcal{E}I = \frac{\mathcal{E}^2}{R}$

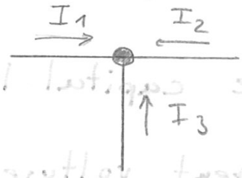
- Kirchhoffs voltage law (KVL)
  - ↳ energy is conserved so around one complete loop charges gain no overall energy



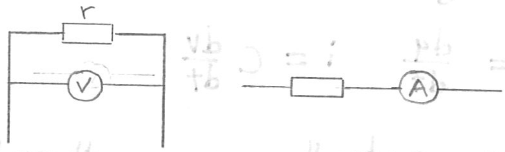
  - ↳ $\sum V = 0$ sum of all pd around complete loop is 0

$\mathcal{E}_1 + \mathcal{E}_2 + V_1 \pm V_2 = 0$

- Kirchhoffs current law (KCL)
  - ↳ charge is conserved so at a junction the sum of all current into the junction is 0
  - ↳ $\sum I = 0$

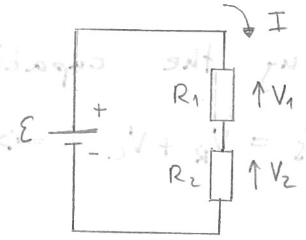


- real measurement devices



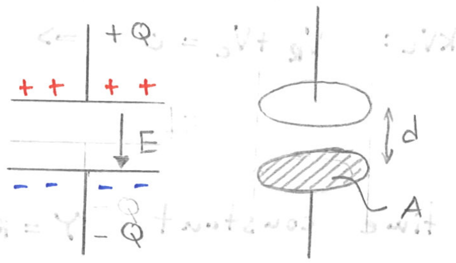
  - ↳ real voltmeter - has parallel resistance $r \approx M\Omega$ ideally $r = \infty$
  - ↳ real ammeter - has series resistance $r \approx m\Omega$ ideally $r = 0$

- equivalent resistance
  - ↳ series resistors $R_{series} = \sum_i R_i$
  - ↳ parallel resistors $R_{parallel} = \dfrac{1}{\sum_i \frac{1}{R_i}}$



- Voltage divider $V_2 = \mathcal{E} \dfrac{R_2}{R_1 + R_2}$

- Capacitance
  - ↳ energy stored due to displaced charge $+Q$ and capacitor remains neutral

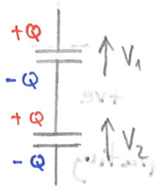


$C = \frac{Q}{V}$ holds for any geometry

- parallel plate capacitor

$C = \dfrac{\varepsilon_0 \varepsilon_r A}{d}$ ε_r - relative permitivity of dielectric between plates

- equivalent capacitance
  - ↳ parallel capacitors $C_{parallel} = \sum_i C_i$
  - ↳ series capacitors $C_{series} = \dfrac{1}{\sum_i \frac{1}{C_i}}$



$Q = C_1 V_1$

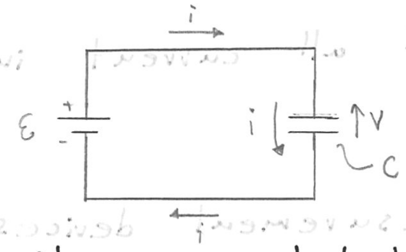
$Q = C_2 V_2$

$V = V_1 + V_2 = \dfrac{Q}{C_1} + \dfrac{Q}{C_2}$

- energy stored in a capacitor
  - during charging $q = CV$ so to increase charge $q \rightarrow q + dq$
    requires energy $du = V dq$ and $V = \frac{q}{C}$

$$du = \frac{q}{C} dq$$
$$\int_0^U du = \int_0^Q \frac{q}{C} dq \Rightarrow U = \frac{1}{2} C Q^2$$

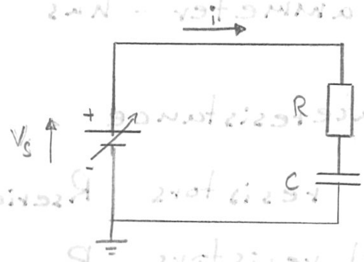
- use capital letters for static quantities and lower case for changing
- current voltage for capacitor



$q = Cv \quad i = \frac{dq}{dt} \quad i = C \frac{dv}{dt}$

  - no charge actually moves through capacitor since dielectric is an insulator. The current is called displacement current

- series RC circuit



  - charging the capacitor: $t = 0 \quad q = 0$

KVL: $V_S = V_R + V_C \Rightarrow \mathcal{E} = iR + \frac{q}{C}$

$$\mathcal{E} = \frac{dq}{dt} R + q \frac{1}{C} \qquad q(t) = C\mathcal{E}\left(1 - e^{-\frac{t}{RC}}\right)$$

  - discharging the capacitor: $t = 0 \quad q = C\mathcal{E}$

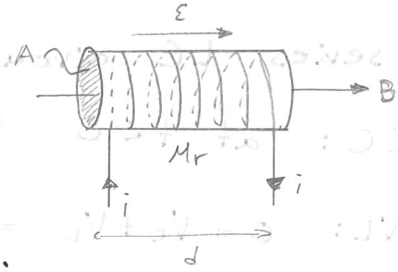
KVL: $V_R + V_C = 0 \Rightarrow 0 = iR + \frac{q}{C}$

$$= \frac{dq}{dt} R + q \frac{1}{C} \qquad q(t) = C\mathcal{E}\, e^{-\frac{t}{RC}}$$

  - time constant $\tau = RC$ after 1 time constant the capacitor will allways charge up / discharge by 63 % of final value $\mathcal{E}$

- power and energy in RC circuit
  - charging: $P_R = i^2 R$ +ve resistor dissipating; $P_C = V_C i$ +ve capacitor charging; $P_S = -V_S i = -\mathcal{E} i$ −ve source delivering E
  - discharging: $P_R = i^2 R$ +ve resistor dissipating; $P_C = V_C i$ −ve capacitor discharging; $P_S = -V_S i = 0$ no source power

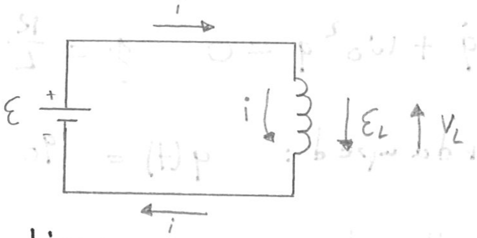
- inductance
  - ↳ changing magnetic flux in coil induces back emf which opposes changes in current



  - ↳ define inductance $L = \frac{\mu_0 \mu_r N^2 A}{d}$ $\quad \mathcal{E} = -L\frac{di}{dt}$

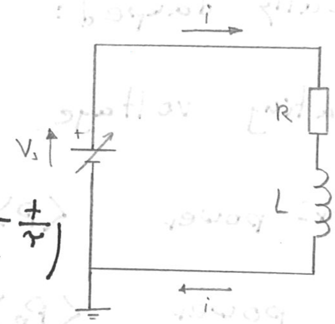
- equivalent inductance
  - ↳ series inductance $L_{series} = \sum_i L_i$
  - ↳ parallel inductance $L_{parallel} = \frac{1}{\sum_i \frac{1}{L_i}}$

- energy stored in an inductor
  - ↳ $P = V_L i$ $\quad \frac{du}{dt} = L i \frac{di}{dt}$ $\Rightarrow$ $U = \frac{1}{2} L I^2$



- current voltage for capacitor $V_L = L\frac{di}{dt}$

- series RL circuit



  - ↳ energising: $t=0$ $\quad i=0$

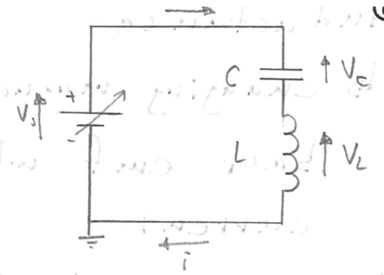
    KVL: $V_s = V_R + V_L \Rightarrow \mathcal{E} = iR + L\frac{di}{dt}$ $\quad i(t) = \frac{\mathcal{E}}{R}\left(1 - e^{-\frac{t}{\tau}}\right)$

  - ↳ de-energizing: $t=0$ $\quad i = \frac{\mathcal{E}}{R}$

    KVL: $0 = V_R + V_L \Rightarrow 0 = iR + L\frac{di}{dt}$ $\quad i(t) = \frac{\mathcal{E}}{R} e^{-\frac{t}{\tau}}$

  - ↳ define time constant $\tau = \frac{L}{R}$

- power and energy in RL circuit
  - ↳ energizing: $P_R = i^2 R$ +ve resistor dissipating $\quad P_L = V_L i$ +ve inductor energising $\quad P_s = -\mathcal{E} i$ -ve source delivering E
  - ↳ de-energizing: $P_R = i^2 R$ +ve resistor dissipating $\quad P_L = V_L i$ -ve inductor releasing E $\quad P_s = -V_s i = 0$ no source power

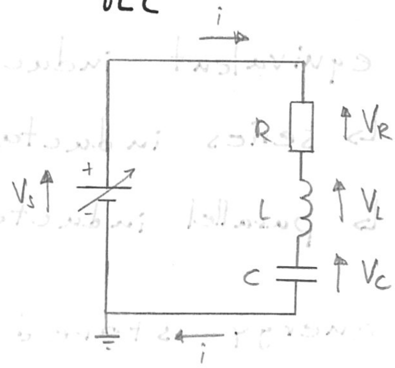


- series LC circuit

IC: at $t<0$ $\quad V_c = \mathcal{E}$ $\quad i=0$ $\qquad$ at $t \geq 0$ $\quad V_c = 0$

kVL: $0 = V_C + V_L \implies 0 = q\frac{1}{C} + L\frac{d^2q}{dt^2}$

gives SHM $\quad q(t) = q_0 \cos(\omega_0 t + \phi) \quad q_0 = C\mathcal{E} \quad \omega_0 = \frac{1}{\sqrt{LC}}$



- damped harmonic oscillator

IC: $V_s = \begin{cases} \mathcal{E} & \text{at } t<0 \\ 0 & \text{at } t \geq 0 \end{cases}$

kVL: $0 = V_R + V_L + V_C \implies \dot{q}R + \ddot{q}L + q\frac{1}{C} = 0$

$\ddot{q} + \gamma\dot{q} + \omega_0^2 q = 0 \quad \gamma = \frac{R}{L} \quad \omega_0^2 = \frac{1}{LC}$

↳ underdamped: $q(t) = q_0 \frac{\omega_0}{\omega_d} e^{-\frac{\gamma t}{2}} \cos(\omega_d t + \phi)$

↳ critically damped: $q(t) = q_0 \left(1 + \frac{\gamma t}{2}\right) e^{-\frac{\gamma t}{2}}$

- alternating voltage $\quad V(t) = V_0 \cos(\omega t + \phi) \quad V_{pp} = 2V_0 \quad V_{RMS} = \frac{V_0}{\sqrt{2}}$

- general AC power $\quad \langle P \rangle = V_{RMS} I_{RMS} \cos(\phi)$

- resistor power $\quad \langle P_R \rangle = \frac{1}{2}\frac{V_0^2}{R}$

- capacitor power $\quad \langle P_C \rangle = 0$ provides no E on avg but phase shifts $\pm\frac{\pi}{2}$

- phasors
  - ↳ amplitude and phase of oscillating quantities represented as complex quantities
  - ↳ $\tilde{Q}$ is a phasor such that $|\tilde{Q}|$ is amplitude and $\arg(\tilde{Q})$ is the phase
  - ↳ don't forget units
  - ↳ kirchhoffs laws also hold for phasors

- complex impedance
  - for any component there is a current $i(t)$ and voltage $v(t)$ oscillating at same ω. write them as $\tilde{I}, \tilde{V}$
  - impedance $\tilde{Z} = \frac{\tilde{V}}{\tilde{I}}$

- impedance of resistor $\tilde{Z}_R = R$
  - real value, no phase change, independent of ω

- impedance of a capacitor $\tilde{Z}_c = -\frac{j}{\omega C}$ phase $-\frac{\pi}{2}$

  $V_c(t) = V_0 \cos(\omega t)$ $\tilde{V}_c = V_0$

  $i(t) = C \frac{dV_c}{dt} = -V_0 \omega C \sin(\omega t) = V_0 \omega C \cos(\omega t + \frac{\pi}{2})$ $\tilde{I} = V_0 \omega C e^{j\frac{\pi}{2}}$

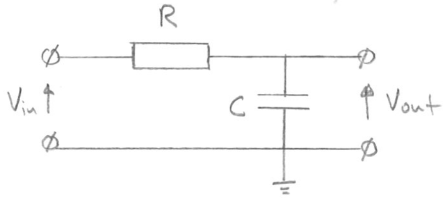
- impedance of an inductor $\tilde{Z}_L = j\omega L$ phase $+\frac{\pi}{2}$

  $i_L(t) = I_0 \cos(\omega t)$ $\tilde{I} = I_0$

  $V_L(t) = L \frac{di}{dt} = -I_0 \omega L \sin(\omega t) = I_0 \omega L \cos(\omega t + \frac{\pi}{2})$ $\tilde{V}_L = j\omega L$

- equivalent impedance
  - series impedance $\tilde{Z}_{series} = \sum_i \tilde{Z}_i$
  - parallel impedance $\tilde{Z}_{parallel} = \frac{1}{\sum_i \frac{1}{Z_i}}$

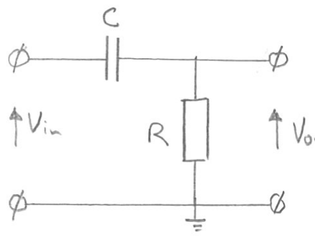
- Low pass RC filter



$\tilde{V}_{out} = \tilde{V}_{in} \frac{\tilde{Z}_c}{\tilde{Z}_R + \tilde{Z}_c} = \tilde{V}_{in} \frac{1}{1 + j\omega CR}$ $\tilde{G} = \frac{\tilde{V}_{out}}{\tilde{V}_{in}}$

$\tilde{G} = \frac{1}{1 + j\frac{\omega}{\omega_c}}$ $G = \frac{1}{\sqrt{1 + \left(\frac{\omega}{\omega_c}\right)^2}}$ $\phi = \arg(\tilde{G}) = -\tan^{-1}\left(\frac{\omega}{\omega_c}\right)$ $\omega_c = \frac{1}{RC}$

- High pass RC filter



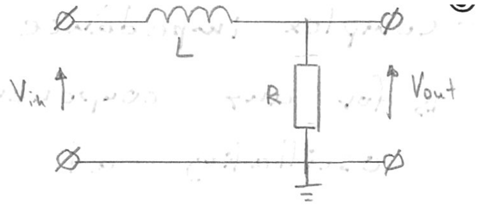
$\tilde{V}_{out} = \tilde{V}_{in} \frac{\tilde{Z}_R}{\tilde{Z}_R + \tilde{Z}_c}$ $G = \frac{\frac{\omega}{\omega_c}}{\sqrt{1 + \left(\frac{\omega}{\omega_c}\right)^2}}$ $\phi = \tan^{-1}\left(\frac{\omega_c}{\omega}\right)$

$\omega_c = \frac{1}{RC}$

- Low pass RL filter

$$\tilde{V}_{out} = \tilde{V}_{in} \frac{\tilde{Z}_R}{\tilde{Z}_R + \tilde{Z}_L} \qquad G = \frac{1}{\sqrt{1 + \left(\frac{\omega}{\omega_c}\right)^2}} \qquad \phi = -\tan^{-1}\left(\frac{\omega}{\omega_c}\right)$$

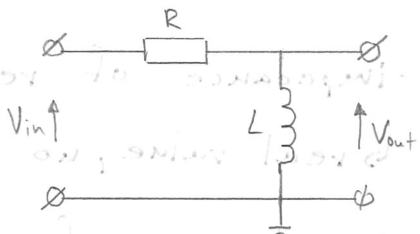
$$\omega_c = \frac{R}{L}$$



- High pass RL filter

$$\tilde{V}_{out} = \tilde{V}_{in} \frac{\tilde{Z}_L}{\tilde{Z}_R + \tilde{Z}_L} \qquad G = \frac{\frac{\omega}{\omega_c}}{\sqrt{1 + \left(\frac{\omega}{\omega_c}\right)^2}} \qquad \phi = \tan^{-1}\left(\frac{\omega_c}{\omega}\right)$$

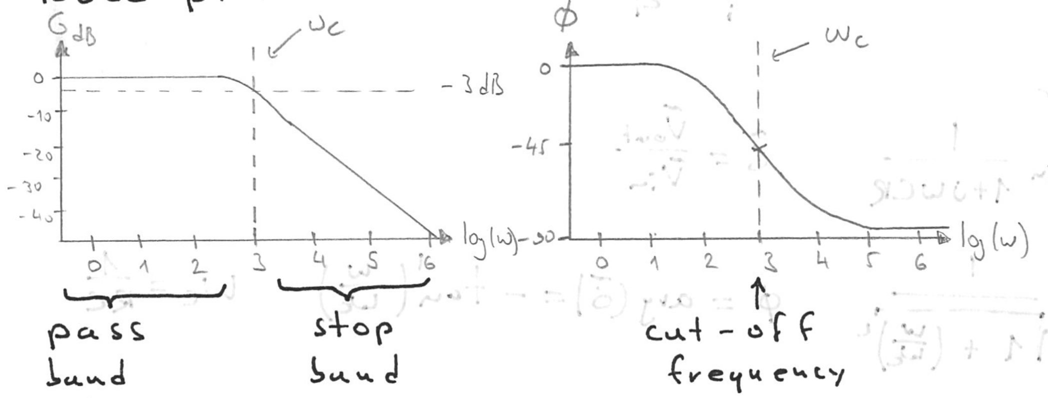
$$\omega_c = \frac{R}{L}$$



- generally:
  - Low pass $\quad G = \frac{1}{\sqrt{1 + \left(\frac{\omega}{\omega_c}\right)^2}} \quad \phi = -\tan^{-1}\left(\frac{\omega}{\omega_c}\right) \quad \omega_c = \frac{1}{RC} = \frac{R}{L}$
  - High pass $\quad G = \frac{\frac{\omega}{\omega_c}}{\sqrt{1 + \left(\frac{\omega}{\omega_c}\right)^2}} \quad \phi = \tan^{-1}\left(\frac{\omega_c}{\omega}\right) \quad \omega_c = \frac{1}{RC} = \frac{R}{L}$

- comments
  - filtered frequencies are dissipated as heat in resistor
  - RC and RL filters have same ω_c they are indistinguishable
  - $\tilde{Z}_C \propto \frac{1}{\omega}$ so cap passes high frequencies
  - $\tilde{Z}_L \propto \omega$ so inductor passes low frequencies

- Bode plot



$$G_{dB} = 20 \log_{10}(G)$$

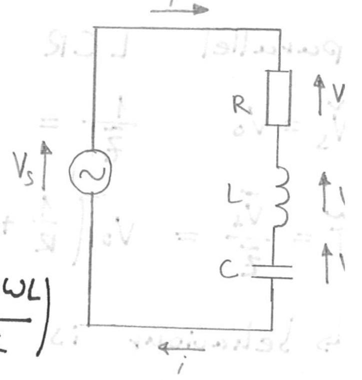
at ω_c $\quad G = \frac{1}{\sqrt{2}}$

$$G_{dB} = -3.01\ldots$$

- series LCR circuit

$V_s(t) = V_0 \cos(\omega t)$ $\quad \tilde{V}_s = V_0$ $\quad \tilde{Z} = \tilde{Z}_R + \tilde{Z}_L + \tilde{Z}_C$

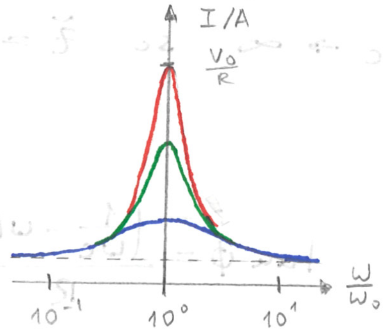
$$= R + j\omega L - \frac{j}{\omega C}$$

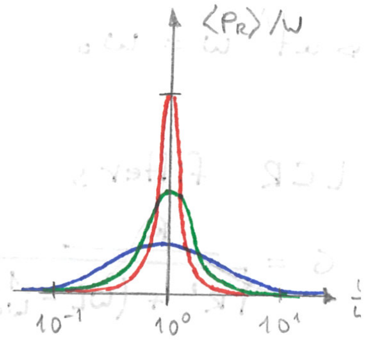


$$\tilde{I} = \frac{\tilde{V}_0}{\tilde{Z}} = \frac{V_0}{R + jL - \frac{j}{\omega C}} \Rightarrow I_0 = \frac{V_0}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}} \qquad \phi = \tan^{-1}\left(\frac{\frac{1}{\omega C} - \omega L}{R}\right)$$

- resonance
  - at resonance $\tilde{Z}_C = -\tilde{Z}_L$ so cancel out and $\tilde{Z} = R$
  - $\tilde{I}$ is maximum $\phi = 0$ and $I_0 = \frac{V_0}{R}$

- at low ω $\quad Z_L$ is $\downarrow$ and $Z_C \uparrow$ so Z overall due to capacitor
  at high ω $\quad Z_L$ is $\uparrow$ and $Z_C \downarrow$ so Z overall due to inductor





- power dissipated in resistor
  - $\langle P_R \rangle = \frac{1}{2} R I_0^2$

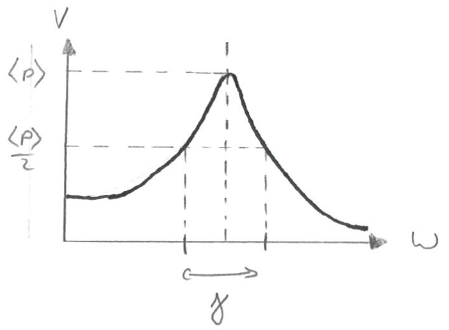
  as R increases damping increases

- Bandwidth - $\Delta\omega$
  - at resonance $\langle P_{res} \rangle = \frac{1}{2} R I_0^2$ now set $\langle P \rangle = \frac{\langle P_{res} \rangle}{2}$

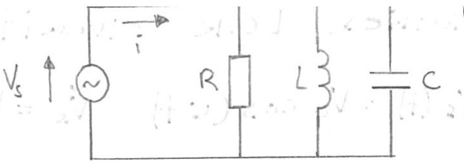
  $\Delta\omega = \frac{R}{L} = \gamma$ $\quad \left(\text{solve } |\tilde{G}| = \frac{1}{\sqrt{2}} \text{ choose } -b + \sqrt{b^2 - 4ac},\ +b + \sqrt{b^2 - 4ac}\right)$

- Q factor
  - dimensionless parameter inversly proportional to strength of damping
  - $Q = \frac{\omega_0}{\Delta\omega} = \frac{\omega_0}{\gamma}$ $\quad$ for LCR: $Q = \frac{1}{R}\sqrt{\frac{L}{C}}$

- note resonance curve is not perfectly symmetric



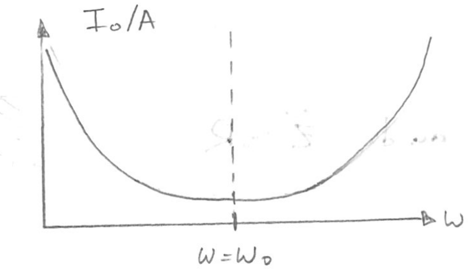
parallel LCR circuit

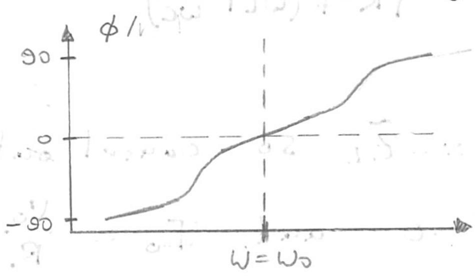


$\tilde{V}_s = V_0 \qquad \frac{1}{\tilde{Z}} = \frac{1}{R} + j\left(\omega C - \frac{1}{\omega L}\right)$

$\tilde{I} = \frac{\tilde{V}_s}{\tilde{Z}} = V_0\left(\frac{1}{R} + j\left(\omega C - \frac{1}{\omega L}\right)\right)$

↳ behaviour is opposite to series LCR giving minimum at $\omega = \omega_0$

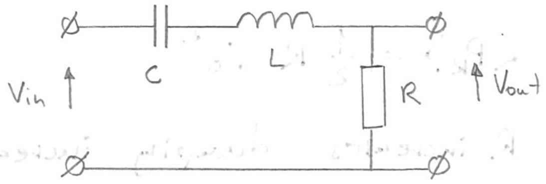




↳ at low ω $\tilde{Z}_L \to 0$ so inductor gives high I
at high ω $\tilde{Z}_C \to 0$ so capacitor gives high I

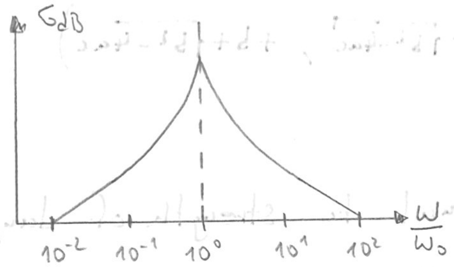
↳ at $\omega = \omega_0$ $\tilde{Z}_{LC} \to \infty$ so $\tilde{Z} = R$

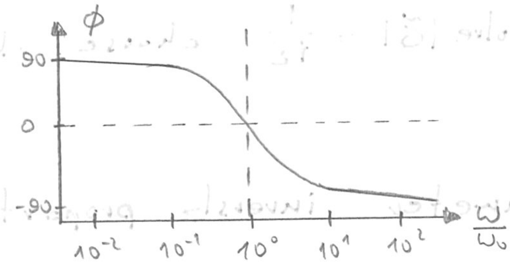
- LCR filters



$G = \frac{R}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}} \qquad \tan\phi = \frac{\left(\frac{1}{\omega C} - \omega L\right)}{R}$

↳ at $\omega = \omega_0$ $G = 1$ $\phi = 0$ $\tilde{Z}_L = -\tilde{Z}_C$
at any $\omega \neq \omega_0$ $\left(\omega L - \frac{1}{\omega C}\right)^2$ becomes greater so G↓





↳ used in analogue radio tuner. air capacitor adjusted to change ω_0