

- radian - angle subtended at the centre of a circle by an arc of a circle whose length is equal to the radius
- if θ small: $\sin \theta \approx \tan \theta \approx \theta$
- angular displacement - angle moved through relative to specific axis
 - $\Delta \theta$ measured in radians
- angular velocity - rate of change of angular displacement
 - ω $[\omega] = \text{rad s}^{-1}$ $\omega = \frac{\Delta \theta}{\Delta t}$ $\omega = \frac{2\pi}{T} = 2\pi f$ $v = r\omega$
- rotational frequency - number of rotations per unit time $f = \frac{n}{t} = \frac{1}{T}$
- centripetal acceleration - $\propto$ $[\alpha] = \text{rad s}^{-2}$ $\alpha = r\omega^2$ $\alpha = \frac{v^2}{r}$
- centripetal force - not actual force - always resultant force which accelerates the object towards centre giving circular motion
 - $F_c = mr\omega^2 = m \frac{v^2}{r}$
- centrifugal force - pseudo force introduced to account for effects of inertia in an accelerated (non-inertial) reference frame
- gravitational field - region of space where a massive object experiences an attractive force
- gravitational field lines - lines indicating direction + strength of gravitational force acting on a test mass in field
 - assume the test mass doesn't affect field itself
- newtons law of gravitation
 - ↳ any two point masses attract each other with a force whose magnitude is directly proportional to the product of their masses and inversely proportional to the square of their separation
 - $F \propto m_1 m_2$ $F \propto \frac{1}{r^2}$ $F_g = \frac{G m_1 m_2}{r^2}$
 - ↳ assuming: masses are point like + separation is much greater than radius
- G - gravitational constant $G = 6.67 \times 10^{-11} \text{ N m}^2 \text{ kg}^{-2}$
- gravitational field strength - g
 - ↳ gravitational force per unit mass at a point in the field
 - ↳ in a radial field: $g = \frac{F_g}{m_{\text{test}}}$ $g = \frac{Gm}{r^2}$ assuming $m_{\text{test}} \ll m$

- gravitational potential energy - E_p

↳ work done in bringing a mass from ∞ to r (point in the field) by gravity

↳ -ve since work is done by force

↳ for radial field where $m_1 \ll m_2$ $E_p = -\frac{Gm_1m_2}{r}$

- gravitational potential - ϕ

$$[\phi] = \text{J kg}^{-1} = \text{m}^2 \text{s}^{-2}$$

↳ at a point is the work done per unit mass in bringing the mass from ∞ to the point

$$\phi = \frac{E_p}{m} \quad W = m_1 \Delta \phi$$

↳ always -ve at $\infty \phi = 0$

for radial field: $\phi = -\frac{Gm_2}{r}$

- equipotential lines

↳ potential doesn't change along them so no work done

↳ perpendicular to field lines if $\vec{S} \cdot \vec{F}_g = 0$ $W = 0$ since $\Delta \phi = 0$

- satellite - object orbiting another object in space

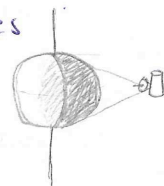
↳ natural: stars, planets, moons ↳ artificial: ISS, weather satellites

- geostationary orbits - $3.6 \times 10^7 \text{ m}$ above equator

↳ can transmit to whole $\frac{1}{2}$ of earth - bad reception at poles

↳ have same ω as earth so appear stationary

↳ rotate west $\rightarrow$ east with earth



- oscillation - repeated movement of system between 2 states (positions) either side of an equilibrium

- amplitude - A - maximum displacement from equilibrium

- frequency - f - number of oscillations per unit time

- period - T - time required for 1 complete oscillation

- phase difference - ϕ - horizontal distance/time which a particular point on a wave leads/lags to another one

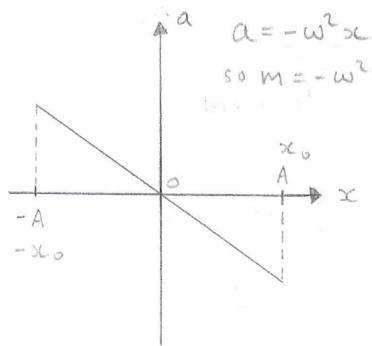
- Simple Harmonic Motion - SHM $a \propto -x$

↳ oscillation in which a is directly proportional and in opposite direction to displacement

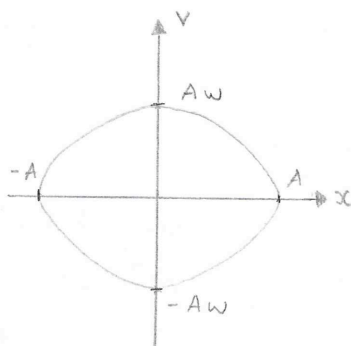
↳ isochronous - period is independent of amplitude

↳ conditions:

- oscillating body
- equilibrium position
- restoring force



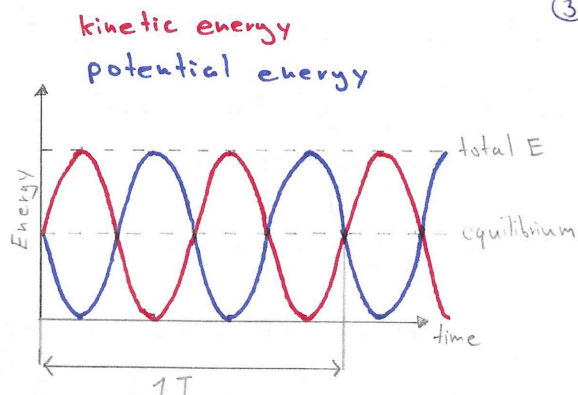
acceleration/displacement



velocity/displacement

$v = x_0 \omega \cos(\omega t)$
 $x = x_0 \sin(\omega t)$

polar equation of ellipse



since KE going down then back up is true in both directions

$T = \frac{1}{f} = \frac{2\pi}{\omega}$ $\omega = 2\pi f$

SHM: $x = x_0 \sin(\omega t + \phi)$
 $v = \omega x_0 \cos(\omega t + \phi)$
 $a = -\omega^2 x_0 \sin(\omega t + \phi)$
 $= -\omega^2 x$

$v = \pm \omega \sqrt{x_0^2 - x^2}$
 $E_{total} = \frac{1}{2} m (\dot{x}^2 + \omega^2 x^2)$

-Damping - reduction in E of oscillation (or Amplitude) due to a force always acting opposite to direction of motion (opposes restoring force)

- exponential decrease in amplitude
- doesn't change period or frequency

-Light damping - gradual decrease in A
- oscillations continue for long

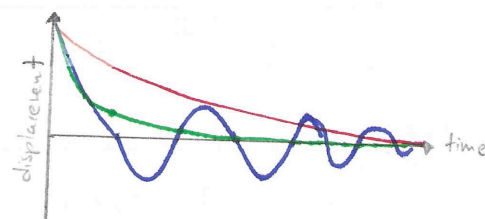
-Critical damping - oscillations stop immediately

-Over damping - amplitude decreases rapidly
- oscillations still occur for short time
- eg spring in viscous oil

$\frac{d^2x}{dt^2} + 2\gamma \frac{dx}{dt} + \omega_0^2 x = 0$

γ - damping factor

gives 3 solution corresponding to each damping scenario



-resonance - when driving frequency = natural frequency resulting in large amplitude $f_{driving} = f_0$

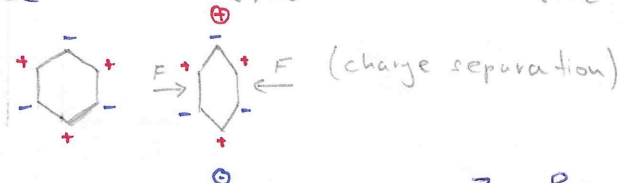
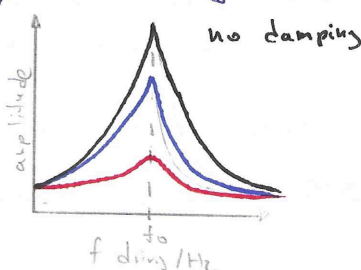
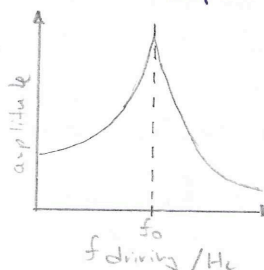
-piezo electric transducer

↳ converts mechanical E ↔ electrical E

↳ thickness should be $\frac{\lambda}{2}$

↳ diameter should be $\approx 10\lambda$ to give nice planar wave (not point source)

↳ resonance at $f_{ac} = f_0$



-acoustic impedance - Z

↳ property of material - how much it opposes acoustic flow

-on boundary some ultrasound is reflected and some transmitted (and refracted).

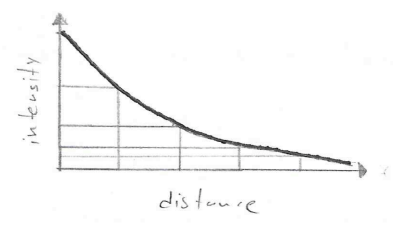
$Z = \rho c$

$[Z] = \text{kg m}^{-2} \text{s}^{-1}$

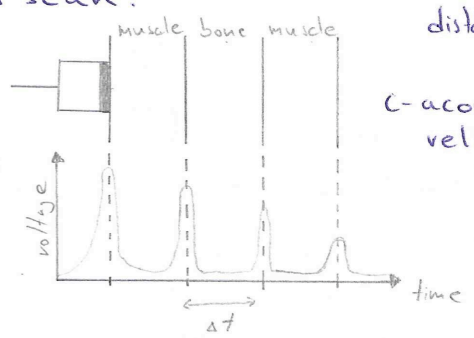
ρ - density

c - acoustic velocity

- intensity reflection coefficient - α $\alpha = \frac{I_R}{I_0} = \left(\frac{Z_2 - Z_1}{Z_2 + Z_1} \right)^2$ (when $\theta_i = 0$)
- coupling gel (medium)
 - ↳ used for impedance matching to $\downarrow \alpha$
- signal attenuated by absorption μ - attenuation coefficient
- half value thickness - $x_{\frac{1}{2}}$
 - ↳ distance a wave travels for transmitted intensity to halve
 - $[\mu] = \text{dbm}^{-1}$ $I = I_0 e^{-\mu x}$
 - $x_{\frac{1}{2}} = \frac{\ln(2)}{\mu}$
- resolution - ability to distinguish 2 separate structures
 - if $f \uparrow, \lambda \downarrow$ resolution $\uparrow$ but penetration $\downarrow$



A scan:



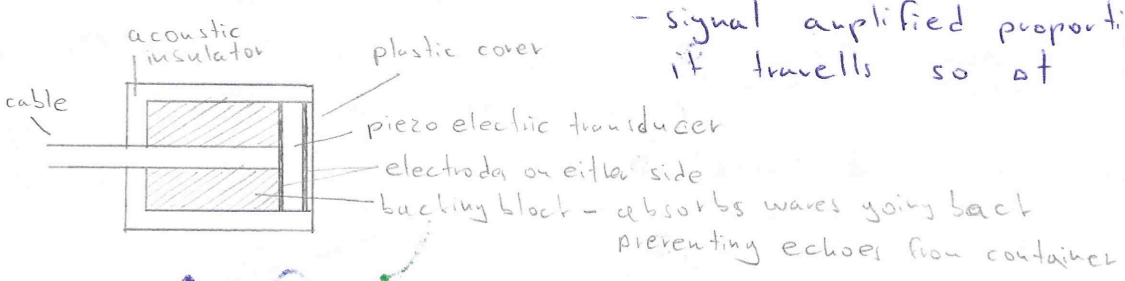
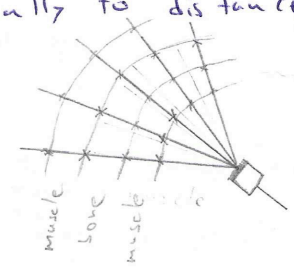
distance = $\frac{C \Delta t}{2}$

C - acoustic velocity

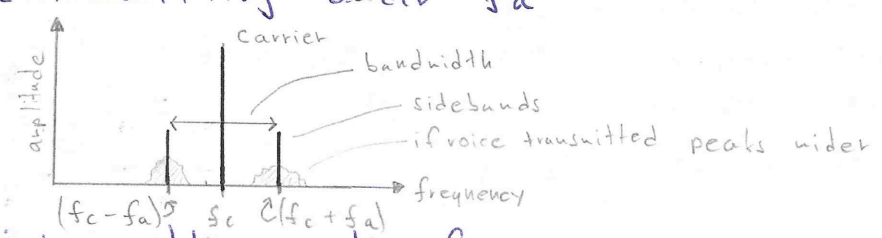
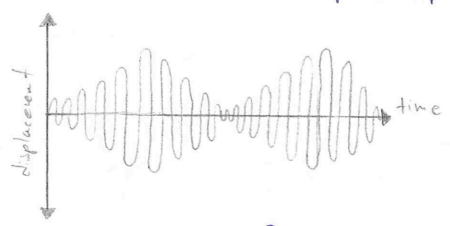
B scan:

↳ series of A scans at different angles plotted on a diagram

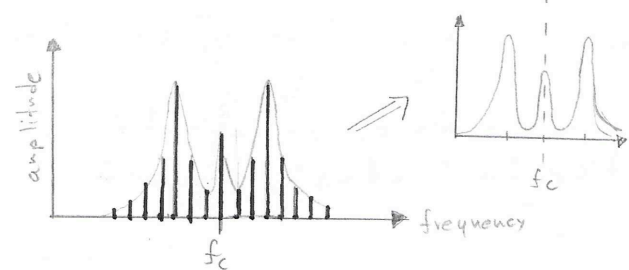
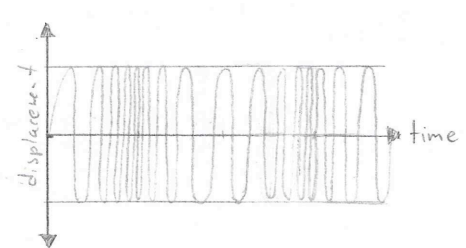
- Δt gives info about distance (thickness) and density of tissue
- amplitude (V) gives info about density
- signal amplified proportionally to distance it travels so at



- AM - amplitude of carrier wave is made to vary synchronously with displacement of transmitted signal.
- FM - frequency of carrier wave is made to vary synchronously with displacement of transmitted signal
- frequency of carrier wave must be at least $2 \times$ frequency of signal
- AM - carrier frequency f_c transmitting audio f_a



- FM - carrier frequency f_c transmitting audio f_a



-AM: advantages

- ↳ cheaper + simpler electronics
- ↳ larger transmission range
∴ covers greater area
- ↳ small bandwidth
∴ more stations per Δf

disadvantages

- ↳ more interference
- ↳ lower bandwidth
∴ lower quality of signal

-FM advantages

- ↳ less electrical interference
- ↳ greater bandwidth ∴ better quality
- ↳ lower transmission range
∴ stations at same f in one area don't interfere
- ↳ higher frequency f_c ∴ greater Δf

disadvantages

- ↳ lower range + more expensive electronics
- ↳ higher bandwidth ∴ less stations per Δf

- attenuation - gradual reduction in signal power

- noise - random unwanted signal added to transmitted signal distorting it

- repeater amplifier - receives signal and transmits it together with the noise at a higher power

- signal to noise ratio stays constant

- regenerator amplifier - receives digital signal, eliminates noise and transmits it at a higher power

- advantages of digital signal:

- ↳ repeater amplifiers can be used ∴ higher range
- ↳ digital circuits cheaper to build and more reliable
- ↳ extra information can be transmitted and used by receiver to check and correct errors in received signal
- ↳ higher rate of transfer of information than analogue

- Binary numbers - each digit is one bit

↳ MSB - most significant bit - on the left

↳ LSB - least significant bit - on the right

↳ telephones use 8 bit $2^8 = 256$ binary numbers

↳ information transmitted right $\rightarrow$ left ($\leftarrow$)

$$6V = \overset{8}{0} \overset{4}{1} \overset{2}{1} \overset{1}{0} \quad 4+2=6$$

MSB $\leftarrow$ $\rightarrow$ LSB

$$13V = \overset{8}{1} \overset{4}{1} \overset{2}{0} \overset{1}{1} \quad 8+4+1=13$$

- ADC - samples analogue signal at regular intervals given by sampling f and sends voltages out as binary data

- rounds down to most significant bit

- DAC - converts binary data into analogue voltage levels

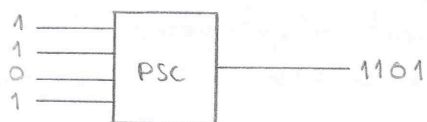
- distorts signal since reproduction isn't accurate (creates steps)

- Nyquist rule - highest frequency which can be reproduced is half the sampling f

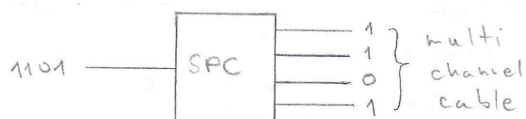
- to reproduce signal f_a $f_{\text{sampling}} \geq 2f_a$

- telephone sampled at 8kHz ∴ can reproduce 3.4 kHz

- parallel to serial converter - takes all simultaneously recieved bits and sends them out as a sequence down a single line



- serial to parallel converter - takes whole sequence of bits and sends them out simultaneously via a multi channel cable



- power falls off logarithmically

- attenuation = $10 \log \left(\frac{P_{in}}{P_{out}} \right)$ in dB gain = $10 \log \left(\frac{P_{out}}{P_{in}} \right)$

- signal to noise = $10 \log \left(\frac{P_{signal}}{P_{noise}} \right)$ in dB attenuation per unit length = $\frac{10}{L} \log \left(\frac{P_{in}}{P_{out}} \right)$ in dBm⁻¹

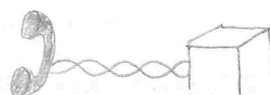
- communication channel - medium or pathway used to convey information between 2 points.

- Wire pairs

↳ signal sent as pd across 2 wires. second wire acts as current return path

↳ used in early land telephones

↳ **X** - repeaters needed - high attenuation



- crosstalk - security wire tapping - wires pick up noise = interference

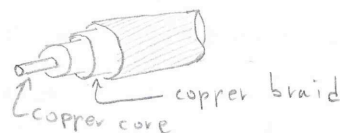
- low bandwidth

- Coaxial cable

↳ signal sent as pd in central copper wire

↳ copper braid - grounded ∴ shielding

- acts as current return path



✓ - better security - less prone to interference + cross talk

- higher bandwidth < 100 MHz - less repeater amplifiers needed

X - more expensive cable

- Radio

frequency multiplexing

↳ different carrier frequencies allow radios to share air space

↳ long range communication using AM long waves reflected off iono sphere

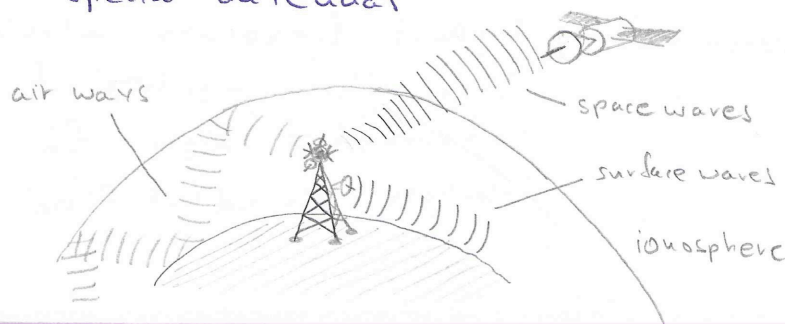
↳ short wave space waves can pass to space

↳ bad reception in valleys - SW don't reflect from ground

↳ for line of sight transmission there must be no obstacles in path

↳ point to point transmission with special antennas

↳ UHF and VHF used in phones + walkie talkies can have very short antennas



- Microwave link

↳ short distance point to point

↳ parabolic reflector focuses E

of the wave into a point or parallel wave. $\downarrow \lambda =$ less spreading out

↳ Satellites - uplink signal: earth $\rightarrow$ satellite downlink: satellite $\rightarrow$ earth

- uplink and downlink have different frequencies to prevent downlink from swamping uplink (results in the feedback)

- 6/4 GHz, 14/11 GHz, 30/20 GHz band-width ≈ 0.5 GHz

↳ satellite orbits

↳ polar - N $\rightarrow$ S $\perp$ to rotation of earth

- scans whole surface

- weather satellites

✓ - low orbit $\therefore$ cheap to launch

✗ - move relative to surface

✓ smaller time delay

↳ geostationary - W $\rightarrow$ E at equatorial orbit

- 36000 km above surface

✓ - one satellite covers almost $\frac{1}{2}$ earth

- stationary to surface so satellite dish can be set permanently

✗ - high orbit $\therefore$ expensive to launch

- large time delay

- bad reception at poles

- Optic fibres

↳ undersea cables for long distance communication (telephone + internet)

↳ digital pulses of visible or IR light created by lasers

✓ - high bandwidth $\approx$ GHz

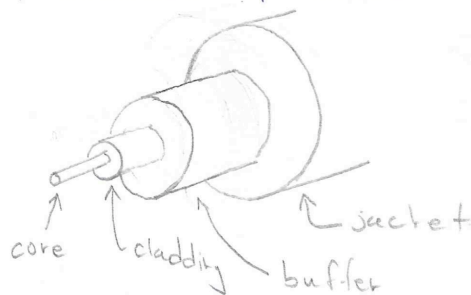
- constant and relatively low noise

- regenerated every 80-200 km

✗ - difficult to connect one another

- expensive technology

- difficult to repair



- electric field - region of space where a charged body experiences a force

- coulombs law - electrical force between 2 charged particles is directly proportional to product of their charges and inversely proportional to square of their separation

- for point charges so radius $\ll$ separation

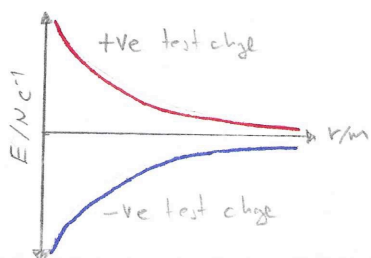
$$F_e = \frac{1}{4\pi\epsilon_0} \frac{Q_1 Q_2}{r^2} \quad -ve = \text{attraction} \quad +ve = \text{repulsion}$$

- ϵ_0 - permittivity of free space $\epsilon_0 \approx 8.85 \times 10^{-12} \text{ C}^2 \text{ N}^{-1} \text{ m}^{-2}$

- electric field strength - force experienced by charged particle in an electric field per unit charge $E = \frac{F_e}{Q}$

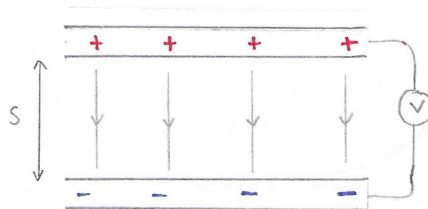
radial:

$$E = \frac{1}{4\pi\epsilon_0} \frac{Q}{r^2}$$



uniform:

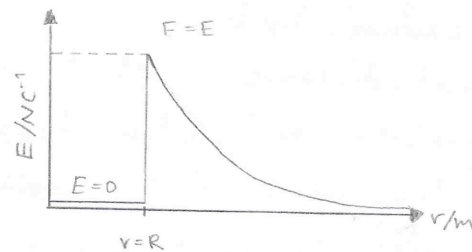
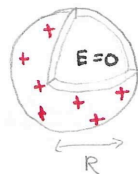
$$E = \frac{V}{s}$$



- electric field of a hollow spherical conductor

↳ inside field cancels out

$$E_{\text{inside}} = 0$$



- electric potential - V

↳ work done per unit charge in bringing it from ∞ to the point in an electric field

$$V = \frac{E_p}{q}$$

- electric potential energy - E_p

radial field: $E_p = \frac{1}{4\pi\epsilon_0} \frac{Qq}{r}$

uniform field: $E_p = Eq$ $E_p = q \int E ds$
 $V = V$

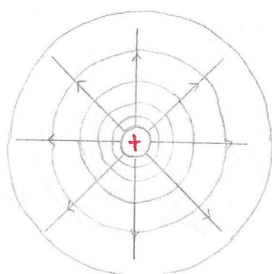
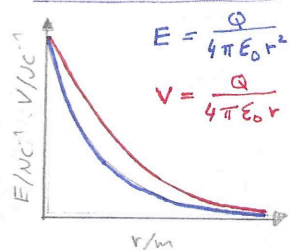
$$V = \frac{1}{4\pi\epsilon_0} \frac{Q}{r}$$

- equipotential lines - lines connecting points of equal potentials (V)
 - perpendicular to field lines

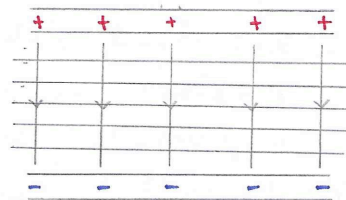
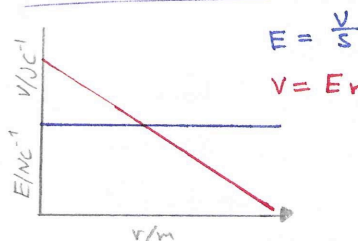
- potential gradient - rate of change of electric potential with

respect to distance in a direction (displacement) within the field
 $\nabla V = -E$ $\left(\frac{\Delta V}{\Delta r} = -E\right)$
 $\nabla V = \frac{dV}{dr}$

- radial field:



- uniform field:

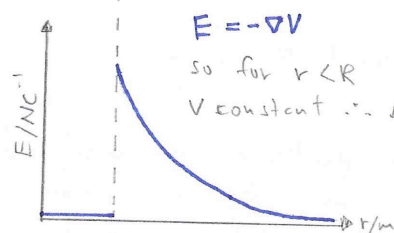
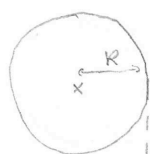


- potential well - region surrounding a local minimum

- E in potential well is unable to do work

- potential hill - region surrounding a local maximum

- E in potential hill able to do work

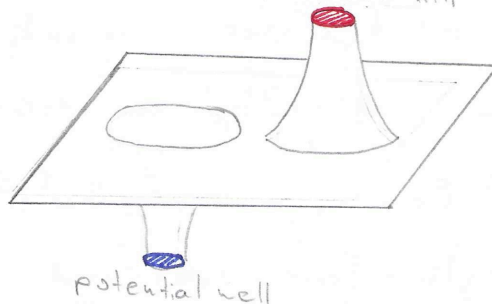


$$E = -\nabla V$$

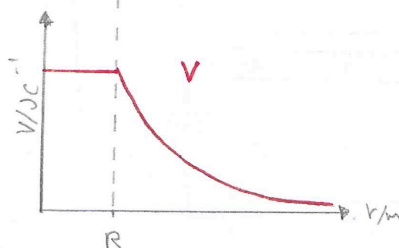
so for $r < R$

V constant $\therefore E = 0$

potential hill



potential well



- Capacitance - charge stored per unit potential difference across it
- Farad - capacitance when pd across capacitor plates is 1V and they store a charge of 1C
- capacitor - always neutral $\therefore$ no net charge
 - doesn't store charge only displaces it to store E

- factors affecting capacitance

↳ area - $C \propto A$

↳ separation - $C \propto \frac{1}{s}$

↳ permittivity of dielectric - $C \propto \epsilon$

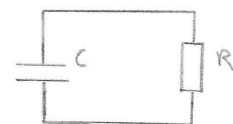
$$C = \frac{\epsilon A}{s}$$

$$C = \frac{Q}{V}$$

parallel: $C_T = \sum_{i=1}^n C_i$

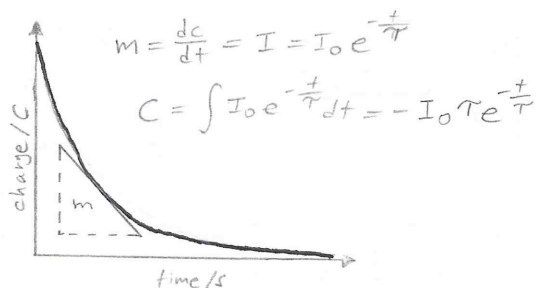
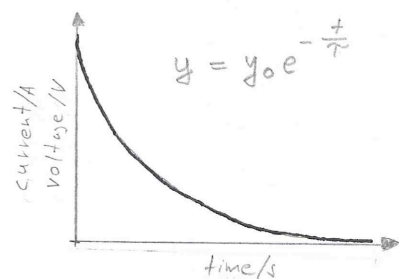
series: $C_T = \frac{1}{\sum_{i=1}^n \frac{1}{C_i}}$

- discharging - time for discharge depends on C and R
 - never actually fully charged or discharged



$\tau = RC$ $V = V_0 e^{-\frac{t}{\tau}}$ $I = I_0 e^{-\frac{t}{\tau}}$ $E = \frac{1}{2} QV = \frac{1}{2} CV^2$

$[\tau] = s$



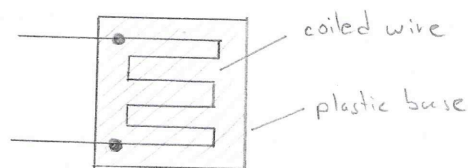
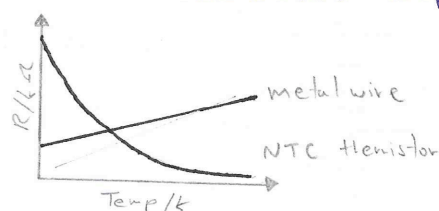
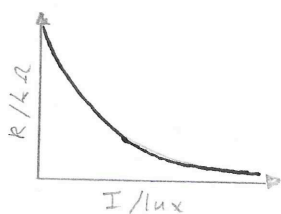
uses:

- ↳ timing - charged/discharged periodically
- ↳ smoothing after rectification
- ↳ tuning of radio - variable C in RC oscillator
- ↳ storing E - camera flash
- ↳ coupling

- electrical sensing system:

Sensing device $\rightarrow$ Processor $\rightarrow$ Output device

- piezoelectric transducer - direct effect: pressure $\rightarrow$ pd or reverse
- LDR - semiconductor - γ absorbed by valence e^- excited into conductive band and $R \downarrow$
 - dark $\approx 1M\Omega$ light $\approx 100\Omega$
- NTC thermistor - semiconductor so same principle as LDR
 - resistance decreases exponentially with $\uparrow T$



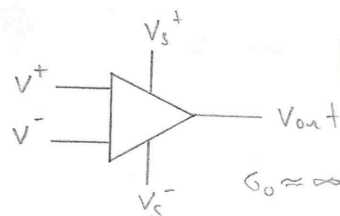
- strain gauge - strain = $\frac{x}{L_0}$ as strain $\uparrow$ $L \uparrow$ and $A \downarrow$ $R = \frac{\rho L}{A}$ so $R \uparrow$
 - stretching $R \uparrow$, compressing $R \downarrow$

$\Delta R \propto \Delta L$

- Operational amplifier

↳ IC containing many transistors arranged to give circuit with very high gain

↳ V^+ - non-inverting input V^- - inverting input
 $P_{out} > P_{in}$ so separate supply needed



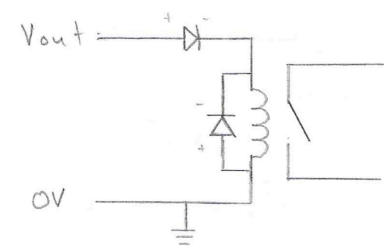
[saturation!]

- ideal OP amp (5)
 - ↳ G_o - open loop gain is ∞
 - ↳ input impedance = ∞ output impedance = 0
 - ↳ bandwidth = ∞ all f signals amplified equally
 - ↳ slew rate = ∞ $\frac{dV_{out}}{dt} = \infty$
 - ↳ 0 noise contribution

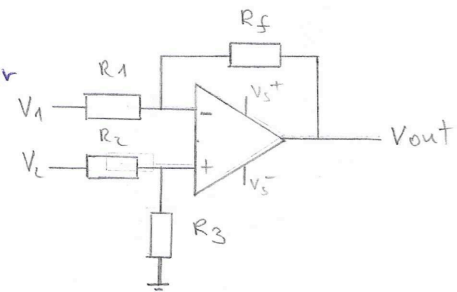
realistically :

$G_o \approx 10^5$
 $R_{in} \approx 10^6 - 10^{12} \Omega$
 $R_{out} \approx 75 \Omega$
 $I_{out} \approx 25 \mu A$
 $V_{out} \approx \pm 15 V$

- use relay to switch higher V or higher A in an external circuit



- differential amplifier

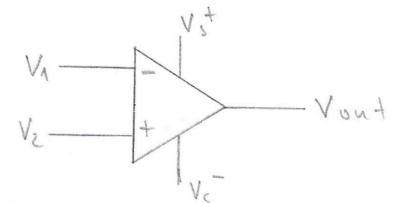


$$V_{out} = \frac{R_f}{R_1} (V_2 - V_1)$$

$$G = \frac{R_f}{R_1}$$

- comparator amplifier

- ↳ saturates
- ↳ 2 types - inverting - non-inverting

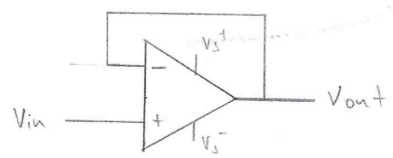


if $V_+ > V_-$ then $V_+ - V_- = +ve$
 so $V_{out} = V_{s+}$

if $V_+ < V_-$ then $V_+ - V_- = -ve$
 so $V_{out} = V_{s-}$

- buffer amplifier

- ↳ entire output fed back to inverting input
- ↳ used for protection + impedance matching e.g. piezo microphone

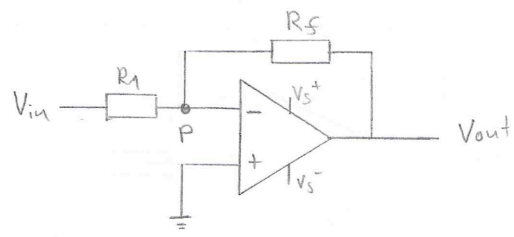


$$G = 1$$

$$V_{out} = V_{in}$$

- Inverting amplifier

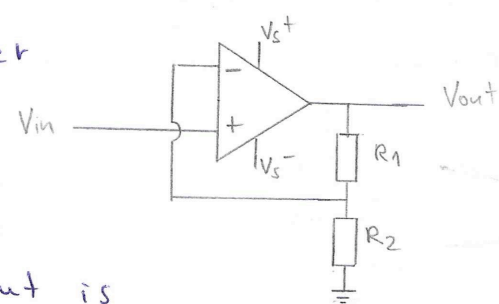
- ↳ P - virtual earth at almost 0V
- ↳ no current flows through opamp



$$V_{out} = - \frac{R_f}{R_1} V_{in}$$

$$G = - \frac{R_f}{R_1}$$

- non-inverting amplifier



$$V_{out} = \left(1 + \frac{R_1}{R_2}\right) V_{in}$$

$$G = 1 + \frac{R_1}{R_2}$$

- feedback

- ↳ fraction of output is added (fed back) to the input

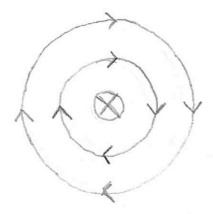
- ↳ advantages: reduced gain, increased bandwidth, increased output stability

- magnetic field $N \rightarrow S$

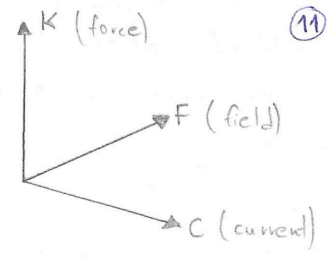
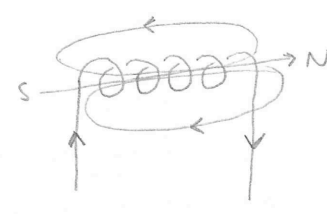
- test pole - north

⊗ - into paper ⊙ - out of paper

- right hand grip rule:



- Fleming's left hand grip rule:



- magnetic flux density - B

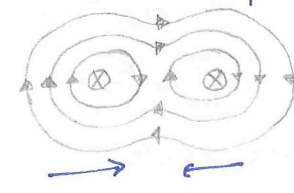
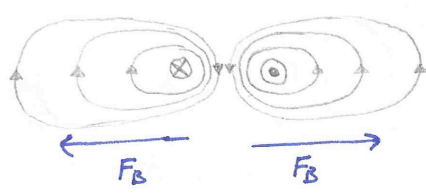
↳ at a point in space is the force experienced per unit length by a long straight conductor carrying a unit current placed at 90° to the field at that point.

$$F = BIL \sin \theta \quad [B] = T = Nm^{-1}A^{-1}$$

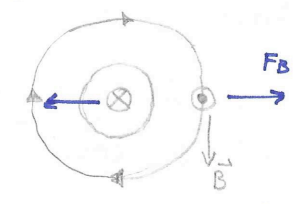
- Tesla

↳ Magnetic flux density of a field when a wire carrying 1A placed at 90° to the field experiences a force of 1N per metre of its length.

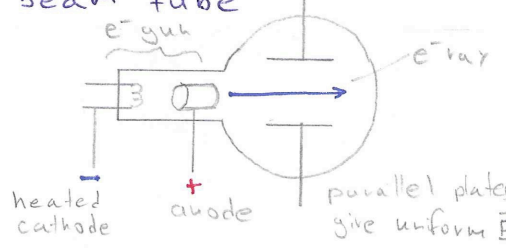
- B measured with: calibrated hall probe or current balance



motor effect:
second force due to Newton's 3rd law of motion



- e- beam tube

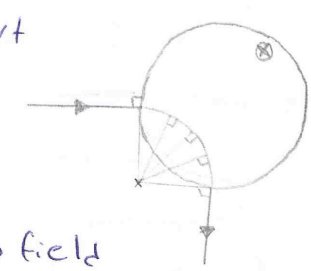


- FB always 90° to $\vec{v}$ so gives centripetal force resulting in circular motion

$$F = BIL \quad I = \frac{Q}{t} \quad L = s = vt$$

$$F = B \frac{Q}{t} vt$$

$$= BQv$$



- if $\vec{v} \cdot \vec{B} \neq 0$ so not perpendicular

Split $\vec{v}$ into components parallel and perpendicular to field and use the one perpendicular to $\vec{B}$ (other one is along equipotential)

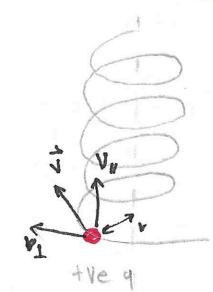
- circular motion:

$$F_B = BQv \quad F_c = m \frac{v^2}{r}$$

$$F_B = F_c$$

$$BQv = m \frac{v^2}{r}$$

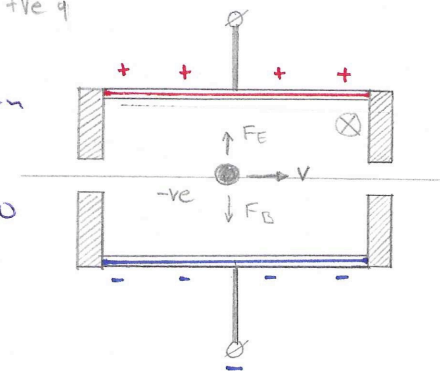
$$r = \frac{mv}{BQ}$$



- charge to mass ratio = $\frac{q}{m}$
- specific charge = $\frac{e}{m_e}$

- Velocity selector

↳ evacuated chamber with uniform electric field $\vec{E}$ and magnetic field $\vec{B}$ at right angles $\vec{E} \cdot \vec{B} = 0$
↳ slit at each end. only particles with given speed can pass



$$F_E = Eq \quad F_B = Bqv$$

$$F_E = F_B$$

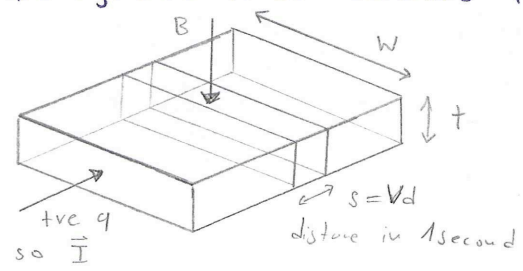
$$Eq = Bqv$$

$$v = \frac{E}{B}$$

invariant of q or m

- Hall probe

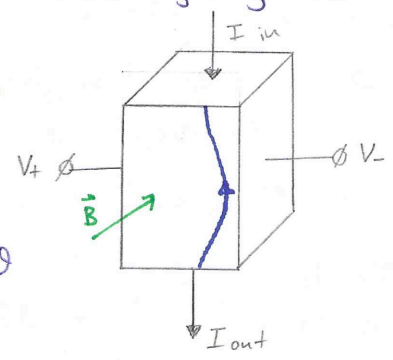
- ↳ small semiconductor strip through which I is passed
- ↳ semiconductor since - e^- travel faster / lower n than metals so greater hall effect per given I
- ↳ magnetic force causes flow of e^- to shift to one side giving V_H



$$F_E = qE \quad F_B = BqV_d \quad I = nWtV_dq$$

$$= q \frac{V_H}{W} \quad = \frac{BI}{nWt} \quad V_d = \frac{I}{nWtq}$$

$$F_E = F_B \quad q \frac{V_H}{W} = \frac{BI}{nWt} \Rightarrow V_H = \frac{BIq}{nt} \sin \theta$$



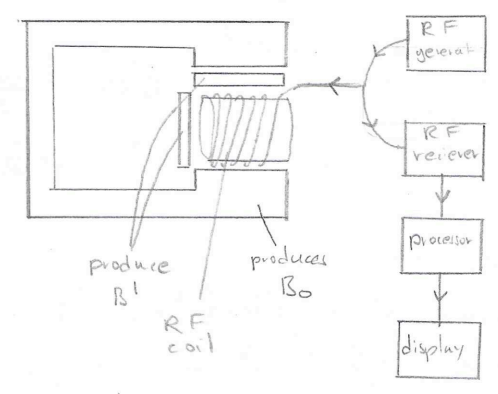
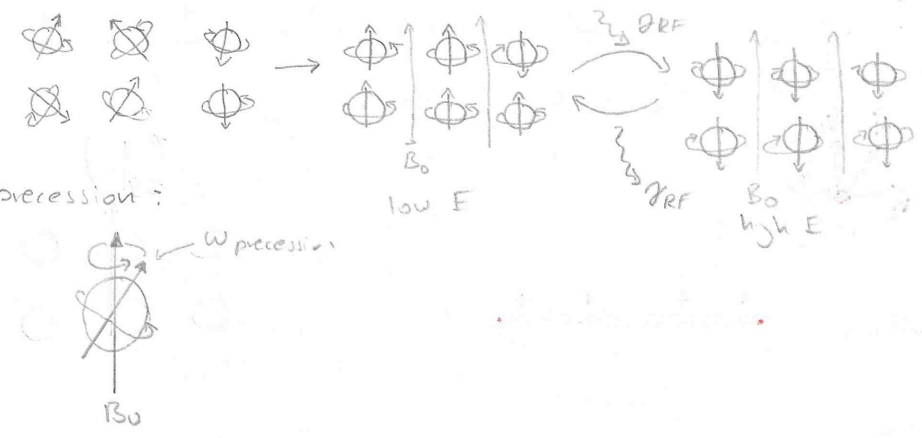
- MRI

- ↳ nuclei have spin $\therefore$ magnetic moment $n^0 + p^+ = \text{odd}$ - half integer spin
- ↳ in body nuclei of H in H_2O n^0 and $p^+ = \text{odd}$ - integer spin
- but aligned randomly so individual $\vec{B}$ cancel out $\Sigma \vec{B} = 0$
- ↳ large $B_0 \approx 1T$ so that f_{Larmor} in RF range

$$f = \frac{\gamma B_0}{2\pi} \approx 42.6 \text{ MHz}$$

for p^+ in $B_0 = 1T$

- 1) strong uniform magnetic field $\vec{B}_0$ aligns nuclei
- 2) nuclei precess about $\vec{B}_0$ with Larmor frequency
- 3) RF pulse at Larmor f applied, resonates with nuclei
- 4) nuclei absorb this E and flip into excited anti-parallel state
- 5) nuclei de-excited by emitting RF pulse in form of RF photon
- 6) RF pulse detected processed and displayed
- 7) calibrated weak non-uniform magnetic field superimposed on $\vec{B}_0$
- 8) adjusts f_{Larmor} of nuclei so only those in one region resonate
- 9) allows for excitation of region to be varied and only one region excited at a time $\therefore$ determines location of nuclei



- Fleming's right hand rule

- magnetic flux - Φ

↳ product of magnetic flux density and area perpendicular to the field

$$\Phi = BA \cos \theta \quad [\Phi] = \text{Weber (Wb)}$$

- magnetic flux linkage - ΦN

↳ measure of magnetic flux of coil $\Phi N = BAN \cos \theta$

- Weber

↳ amount of magnetic flux with density 1T passing through an area of 1m^2 perpendicular to the field

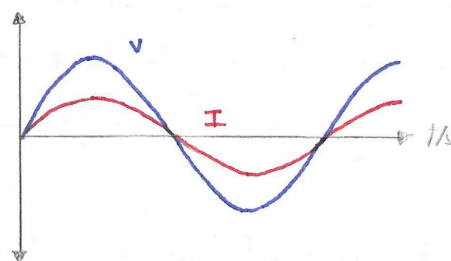
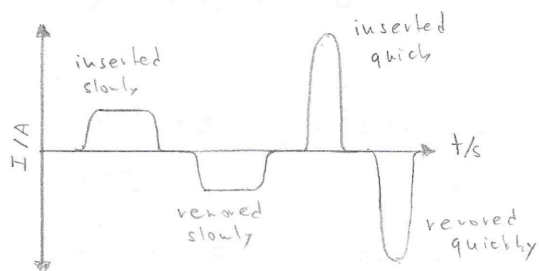
- Faraday's law of EM induction

↳ magnitude of induced emf is equal to the rate of change of magnetic flux linkage through a circuit

$$\mathcal{E} = -N \frac{d\Phi}{dt} \approx -N \frac{\Delta\Phi}{\Delta t}$$

- General Lenz's Law

↳ induced current is established in a direction so as to produce effects which oppose the change causing it



for $R > 1$

- induced I depends on induced $\mathcal{E}$ and R of the circuit

- as magnet falls through coil while it is fully inside $\mathcal{E} = 0$

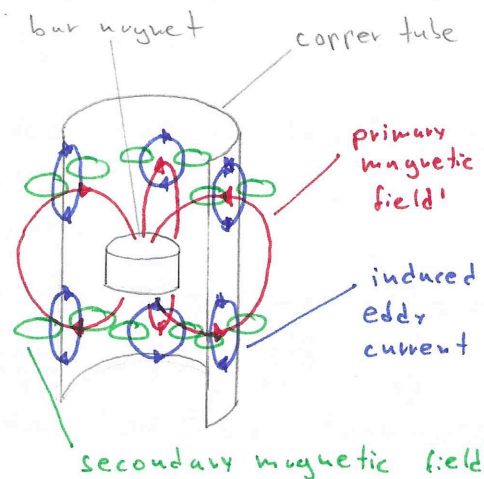
- Eddy currents

1) as magnet falls there is $\frac{d\Phi}{dt}$ of magnetic flux linkage in the copper tube

2) eddy current induced in the conductor

3) eddy current = flow of e^- so produces a secondary magnetic field

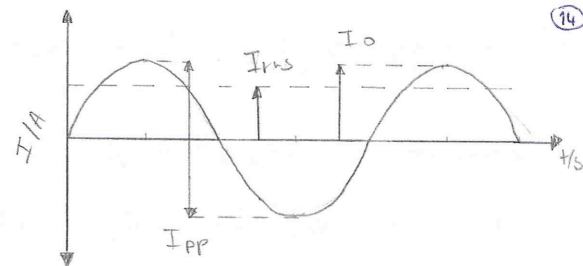
4) 2^o magnetic field opposes motion of magnet by Lenz's law so force acts against gravity and $a < g$



- ω - angular frequency $\omega = 2\pi f$
- I_0 - peak current
- I - momentary current
- I_{pp} - peak to peak current $I_{pp} = 2I_0$

$$V = V_0 \sin(\omega t)$$

$$I = I_0 \sin(\omega t)$$



- RMS current I_{RMS}

↳ steady DC which delivers the same average power to a resistive load as the AC $I_{RMS} = \frac{I_0}{\sqrt{2}}$

↳ AE both t_1 - so avg would be 0 therefore square it then average $I \rightarrow I^2 \rightarrow \frac{I^2}{2}$ and square root to $\frac{I}{\sqrt{2}}$

- Transformer - device which changes peak value of alternating voltage using principle of EM induction

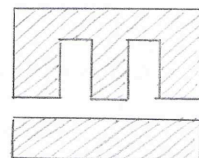
- 1) I in 1^o coil magnetizes soft iron core producing a changing magnetic flux
- 2) magnetic flux in core changes magnitude + direction continuously
- 3) magnetic flux is linking with the 2^o coil
- 4) by faradays law of EM induction $\mathcal{E}$ is induced in 2^o coil

- Eddy currents

- ↳ Alternating field in core is different at different points so $\mathcal{E}$ induced in each point is different
- ↳ current flows between regions, P lost to heating effect

- Loss in transformer

- ↳ resistive losses - resistance of wires $P = I^2 R$ - use low R wires
- ↳ flux losses - magnetic flux from 1^o doesn't completely link with 2^o
 - 2^o coil is wound on top of 1^o $\therefore$ good coupling
- ↳ hysteresis losses - constantly magnetised/demagnetised 100 times s^{-1} and resists it
 - use soft iron core



- ↳ eddy currents - laminated core used

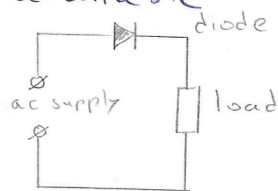
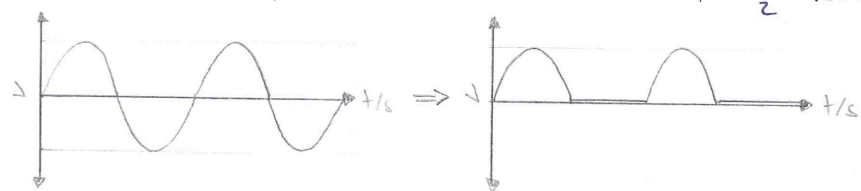
$$\frac{V_2}{V_1} = \frac{N_2}{N_1} \quad \eta = \frac{P_{out}}{P_{in}} = \frac{V_2 I_2}{V_1 I_1} \quad \eta = 1 \quad \frac{V_2}{V_1} = \frac{I_1}{I_2}$$

- National grid - network of power lines transmitting electrical energy

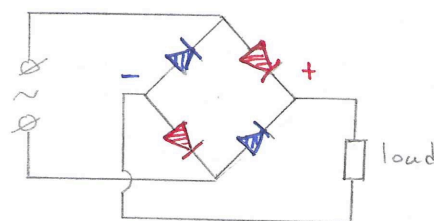
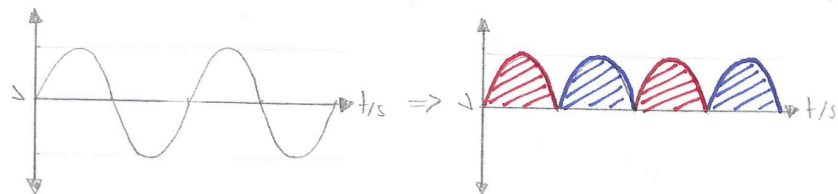
- ↳ AC - required by transformers used to step up/down voltage $\therefore$ varied
 - easily converted to DC

- ↳ High voltage - reduces losses to heating in power lines at a constant P $I \downarrow$ so $P_{lost} = I^2 R$

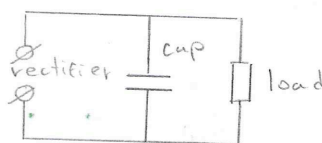
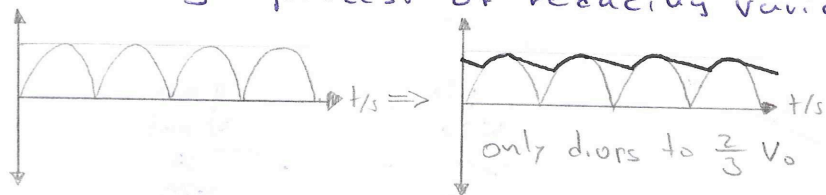
- diode - semi conductor - allows I to only flow in 1 direction
- Rectification - process which makes AC flow in only 1 direction
 - ↳ Half wave rectification - only $\frac{1}{2} P_{rms}$ available



- ↳ Full wave rectification - all of P_{rms} available



- Smoothing - process of reducing variation in rectified voltage



- ↳ to reduce ripple: - $\uparrow C$ of capacitor

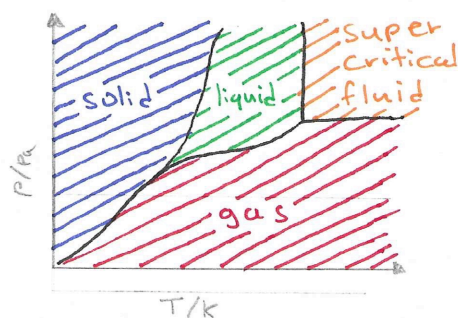
- $\uparrow R$ of load so smaller I slower discharge of C

$$V_{ripple} \propto RC$$

- Thermocouple 
 - ✓ - self powered, wide range, responsive, cheap
 - ✗ - non-linear, low voltage (amplifier), low sensitivity
- RTD 
 - ✓ - most stable, most accurate, most linear
 - ✗ - expensive, self heating, current source required
- Thermistor 
 - ✓ - high output V , only 2 wires, measured in ohms
 - ✗ - non-linear, limited range, fragile, self heating

- thermodynamic temperature - k $T_{(K)} = \Theta_c + 273.15$
- $0k$ - absolute zero - temp at which particles have $0 E_k$

- triple point of H_2O - $273.16k$ $6.12 Pa$
 - ↳ point at which pure H_2O exists in a thermodynamic equilibrium with ice, water, vapour



- Thermal equilibrium

↳ two systems at the same temperature and there is no net flow of thermal energy between them

- Thermal capacity - C

↳ amount of thermal energy required to raise temp of body by 1K

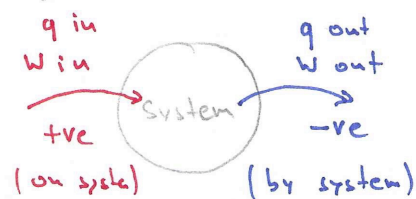
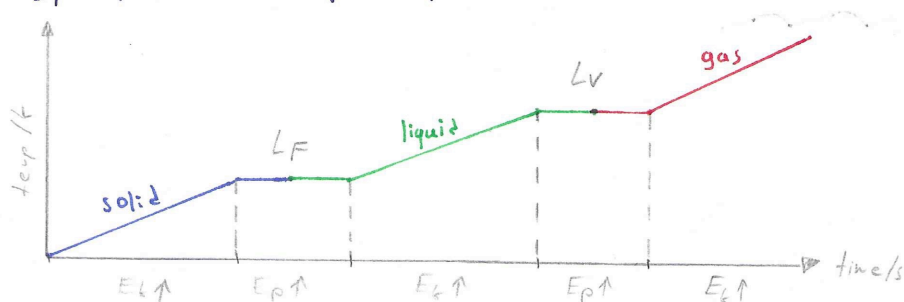
- Specific thermal capacity - c

↳ amount of thermal energy required to raise temp of body of mass 1kg by 1K (unit mass)

- Specific latent heat - L

↳ amount of thermal energy required to change state of unit mass of pure substance without a ΔT

L_F - fusion L_V - vaporisation



- Thermal energy - Q

↳ component of system's internal E due to its temp, random distribution of translational E_k of particles.

- Heat - net flow of thermal energy

- Internal energy - U

↳ sum of random distribution of translational kinetic and potential energies

- 1st law of thermodynamics

↳ formal statement of conservation of E

$$\Delta U = q + W$$

- pressure

↳ normal force exerted per unit area by gas on walls of container

- volume

↳ measure of space occupied by gas

- Brownian motion

↳ random motion of massive particles due to collisions with smaller lighter particles

- Amount of substance - mol - n
 - ↳ amount of substance which contains same number of particles as there are atoms in 0.012 kg of C-12 isotope
- molar mass - M
 - ↳ mass of 1 mole of substance $n = \frac{N}{N_A} = \frac{m}{M}$
- avogadro constant - N_A $N_A = 6.02 \times 10^{23}$
- avogadro's law $\frac{V}{n} = k$
 - ↳ same vol of different gases at same temp and pressure will contain equal no of particles

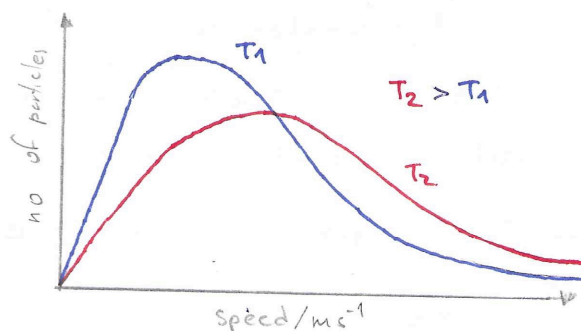
- Ideal gas law - only for ideal gases

$PV = nRT$ $R = 8.31 \text{ J mol}^{-1} \text{ K}^{-1}$
molar gas constant

$PV = NkT$ $k = 1.38 \times 10^{-23} \text{ J K}^{-1}$
Boltzmann constant

- ↳ assumed ideal at $\downarrow P$ and $\uparrow T$

- Maxwell Boltzmann distribution
 - ↳ Area under curve = no of particles
 - ∴ Area stays constant



- root mean square speed - C_{rms}

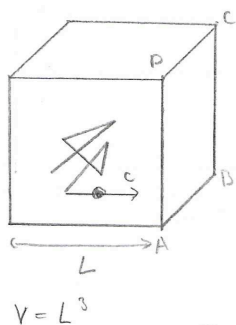
- mean square speed - $\langle c^2 \rangle$

$$C_{rms} = \sqrt{\frac{\sum_{i=1}^N (c_i)^2}{N}} \quad \langle c^2 \rangle = (C_{rms})^2 = \frac{\sum_{i=1}^N (c_i)^2}{N}$$

- assumptions

- ↳ particles move with random + continuous motion
- ↳ particles have negligible vol compared to gas (container)
- ↳ all collisions perfectly elastic
- ↳ duration of collisions $\ll$ time between collisions
- ↳ no potential energy of particles - no forces between them

- mean translational kinetic energy of a particle or ideal gas is proportional to the thermodynamic temperature



$t = 2 \frac{L}{c}$ $\Delta p = -2mc$

$F = \frac{\Delta p}{\Delta t}$
 $= \frac{-2mc}{2 \frac{L}{c}}$
 $= -\frac{mc^2}{L}$

$P = \frac{F}{A} = \frac{mc^2}{V}$

$\langle c^2 \rangle = c_x^2 + c_y^2 + c_z^2$
 $c^2 = \frac{1}{3} \langle c^2 \rangle$

$P = \frac{1}{3} \frac{Nm \langle c^2 \rangle}{V}$
 $= \frac{1}{3} \rho \langle c^2 \rangle$

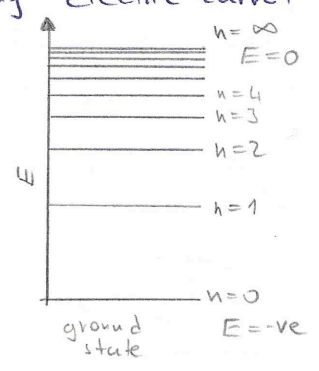
$PV = \frac{1}{3} Nm \langle c^2 \rangle$

$\langle E_k \rangle = \frac{3}{2} nRT$ - for gas

$\langle E_k \rangle = \frac{3}{2} kT$ - for particle

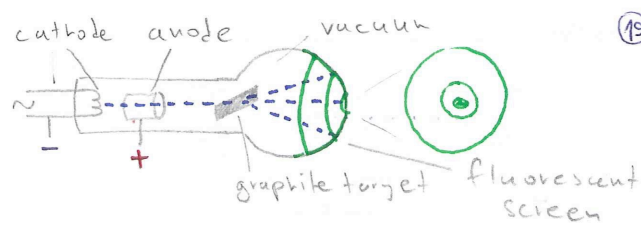
$\langle E_k \rangle \propto T$ $C_{rms} \propto \sqrt{T}$

- photon - discrete quantized packet of EM radiation energy
- proof of wave nature - diffraction + interference
- proof of particulate nature - photoelectric effect
- electron volt
 - ↳ amount of E transferred to single e^- accelerated through pd of 1V
- threshold frequency - f_0
 - ↳ minimum frequency of photons required to release e^- from metal surface
- work function energy - ϕ
 - ↳ minim E required to for an e^- to escape metal surface ($E_f = 0$)
- $\phi_{\text{metal}} \approx 10^{-19}$ $\lambda = \frac{c}{f}$ $E_f = hf = \frac{hc}{\lambda}$ $E_f = \phi + E_k$
 $hf = hf_0 + \frac{1}{2} m_e v^2$
- observations
 - ↳ Threshold f - photoelectric effect only occurs if f above certain value
 - ↳ Time of emission - e^- emitted instantaneously or never if $f < f_0$
 - ↳ Intensity - greater intensity greater no of e^- emitted per time if $f > f_0$
 - ↳ Max E_k of photoelectrons - only depends on f of light
- photoelectric current - I_p
 - ↳ rate of emission of e^- $I_p \propto I$ $I_p \propto \frac{1}{f}$ at constant P
- max E_k of photoelectrons
 - ↳ independent of intensity
- ionisation energy
 - ↳ amount of E required to remove a valence e^- of isolated gaseous atom to form isolated gaseous ion
- causes of ionisation - nuclear radiation, heating, passing electric current
- excitation
 - ↳ promotion of e^- to higher E levels within atom
 - ↳ incoming e^- collides with valence e^- transferring E_k to excite the valence e^-
 - ↳ incoming γ absorbed by e^- and its E excites e^- occurs only if $E_f = \Delta E$



- line spectra - only in vapours / gases at low P
 - γ re-emitted in all directions
- absorption spectra - white light passed through cool gas

- De Broglie wavelength $\lambda = \frac{h}{p}$
 ↳ longer wavelength less particle like

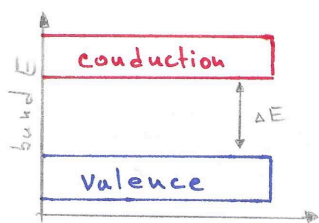


- graphite target acts as diffraction grating
 in 2 planes $\therefore$ circular pattern

- if anode-cathode voltage $\uparrow$ speed of $e^- \uparrow$ so $\lambda \downarrow \therefore$ less diffraction

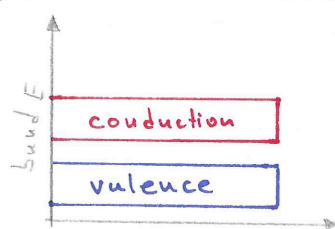
$$\lambda = 2d \sin \theta \quad d - \text{lattice spacing}$$

- bands created from line spectra since bonding in solids affects ΔE
 and spreads out spectral line into band



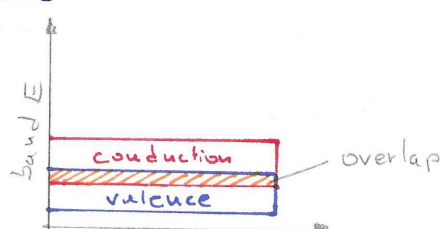
insulator

valence band full
 large ΔE



semi-conductor

valence band full
 small ΔE



conductor

conduction band already
 partially filled

- resistance of metals

↳ as $T \uparrow$ rate of collision of e^- with ions in lattice $\uparrow$ so resistance slightly increases $v_d \downarrow \quad I = N A v_d q \quad I \downarrow \quad R = \frac{V}{I} \quad R \uparrow$

- resistance of semi conductors

↳ as $T \uparrow$ e^- absorb thermal E and jump to conduction band + leave behind +ve holes. both e^- + holes act as charge carriers so resistance significantly $\downarrow$. $\uparrow$ in collision is insignificant $N \uparrow$

- Brehmstrahlung radiation - from deceleration of charged particles
 ↳ range of decelerations of e^- so gives continuous distribution of λ of photons produced

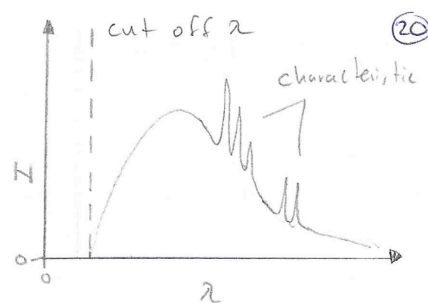
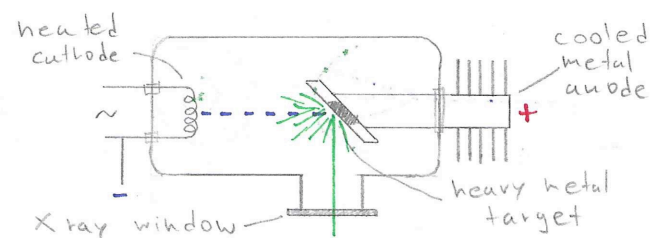
- cut off wavelength - highest E photon when all E_k of e^- goes to E of single produced photon

$$W = qV \quad E_f = \frac{hc}{\lambda} \quad \lambda = \frac{hc}{qV}$$

- characteristic radiation - e^- from beam collide with valence e^- giving them E_k to excite them

- e^- de-excite and emit γ giving line spectrum

- X rays emitted in all directions + 1% input power goes to X rays so rest is lost as heat



- cooling anode - copper good heat conductor
 - fins to ↑ SA + black surface + water cooling
 - anode spun to ↑ target area
- intensity - (mA) - tube current - hardness - (kV) - tube voltage
- Attenuation of X ray photons
 - ↳ some γ absorbed but individual E_γ constant
- Half value thickness - $x_{\frac{1}{2}}$
 - ↳ distance in medium which wave has to travel for transmitted intensity to decrease to $\frac{1}{2}$ original
- sharpness - good definition of edges between 2 body parts (features)
 - ↳ limiting aperture (overlapping metal plates)
 - ↳ small area of target anode
- contrast - difference in degree of blackening between neighbouring features
 - ↳ collimator grid - only transmits normal rays
 - ↳ contrast medium - e.g. barium meal
- reduce exposure
 - ↳ aluminium filter across window - filters out soft X rays
 - ↳ high sensitivity detector or fluorescent backing
- voxel - 3 dimensional imaginary cubes
- ray sum - resultant attenuation of X ray beam passing through all voxels along a line
- background intensity - sum of readings in each detector position
- CT scan
 - ↳ ✓ better detail, 3D, most organs visible
 - ↳ ✗ requires no motion, high dose, risk of childhood cancer if pregnant

$$I = I_0 e^{-\mu x}$$

$$x_{\frac{1}{2}} = \frac{\ln 2}{\mu}$$

- principles of CT

- 1.) X ray images taken of crosssection in one plane
- 2.) repeated at different angles in same plane
- 3.) data recorded + processed
- 4.) detailed 2D image of each crosssection produced
- 5.) repeated for successive crosssections (planes)
- 6.) 3D image build up
- 7.) final image viewed from different angles / rotated
- 8.) computer used to process + store large sets of data

- total mass + energy of isolated system is constant

- mass of nucleus always < total mass of nucleons making it up

- mass defect - Δm

↳ difference between mass of nucleus and total mass of its nucleons

$$\Delta m = \sum M_{\text{nucleons}} - M_{\text{nucleus}} \quad E = mc^2$$

- binding energy - E_B

↳ energy required to separate all nucleons of nucleus to ∞

$$E_B = \Delta mc^2 \quad \text{in MeV} \quad E_B = \Delta m \times 934$$

$$E_{BN} = \frac{E_B}{A}$$

- binding energy per nucleon - E_{BN}

↳ indicates how tightly nucleons are bonded on average

- maximum E_B per nucleon at Fe-56

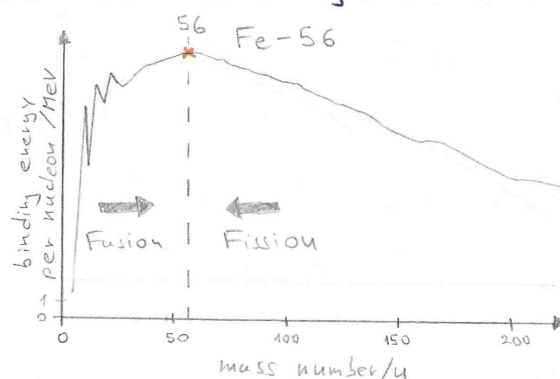
↳ most stable and potential well is deepest

$$E_{BN} \approx 8.818 \text{ MeV}$$

- mass excess

↳ $M_{\text{nucleus}} - \text{nucleon number } (A)$

↳ if +ve has more E_B per nucleon than C-12 likely to be unstable



- Fission - splitting of heavy unstable nuclei into lighter approximate same size nuclei

↳ spontaneous or induced

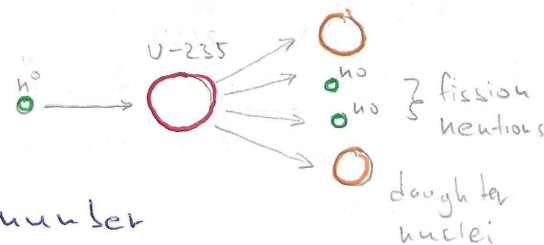
- Fusion - combining of 2 lighter nuclei to form a single heavier nucleus

↳ occurs only if nuclei have sufficient E to overcome electrostatic repulsion to get close enough to be attracted by strong nuclear force

- reaction energetically feasible if $E_B \text{ products} > E_B \text{ reactants}$

↳ energy released given as $-ve \Delta E_B$ or $(m_{\text{products}} - m_{\text{reactants}})c^2$

- smallest amount of mass required to reach chain reaction - critical mass



- A - nucleon number Z - proton number N - neutron number

- radioactive decay

↳ random disintegration of unstable nucleus by emission of particles or photon (EM E) to give more stable nucleus

- activity - A

↳ number of unstable radioactive nuclei which decay per unit time $[A] = \text{Becquerel (Bq)}$

↳ depends on: decay constant - λ , no of unstable nuclei - N

- decay constant - λ

↳ constant probability of decay per unit time

↳ remains constant

$$A = \lambda N$$

$$A = - \frac{dN}{dt}$$

- random decay

↳ constant P per unit time of nucleus but can't determine which particular nucleus decays next

- spontaneous decay

↳ unaffected by presence of other nuclei or environment / conditions
eg: chem reaction, temp, pressure

- half life - $t_{1/2}$

↳ time taken for no of radioactive particles in sample to halve from initial value

↳ time taken for activity of radioactive sample to halve from original value

$$N = N_0 e^{-\lambda t} \quad A = A_0 e^{-\lambda t} \quad t_{1/2} = \frac{\ln 2}{\lambda}$$