Classical Mechanics

- general equations for counterclockwise rotation $x(t) = R\cos(wt)$, $y(t) = R\sin(wt)$ w - angular frequency $w = \frac{2\pi}{T}$

- Lagrangian: action of trajectory $A = \int_{t_0}^{t_1} (T-V)\,dt$ T - kinetic energy, V - potential energy

$L = T - V$ euler-lagrange $\frac{d}{dt}\frac{\partial L}{\partial \dot{x}} - \frac{\partial L}{\partial x} = 0$ or $\frac{d}{dt}\frac{\partial L}{\partial \dot{x}} = \frac{\partial L}{\partial x}$

$p_i = \frac{\partial L}{\partial \dot{q}_i}$ $F_i = -\frac{\partial V}{\partial q_i}$ $L = L(q_i, \dot{q}_i, t)$

p_i - canonical momentum

- Hamiltonian: if system is time translation invariant H conserved

$H = T + V$ $\dot{p}_i = -\frac{\partial H}{\partial q_i}$ $\dot{q}_i = \frac{\partial H}{\partial p_i}$ $H = H(q_i, p_i)$

gibbs louiville theorem - divergence of phase space of H is 0

- $\vec{\nabla} \cdot \vec{V} = \sum_i \left(\frac{\partial}{\partial q_i}\frac{\partial H}{\partial p_i} - \frac{\partial}{\partial p_i}\frac{\partial H}{\partial q_i} \right) = 0$
- phase space of H 2N dimentional (p, q for every N)

- Poission brackets - $\dot{F} = \{F, H\} = \sum_i \left(\frac{\partial F}{\partial q_i}\frac{\partial H}{\partial p_i} - \frac{\partial F}{\partial p_i}\frac{\partial H}{\partial q_i} \right)$ also $\dot{q}_i = \{q_i, H\}$, $\dot{p}_i = \{p_i, H\}$

definition + axioms:

$$\{A, C\} = \sum_i \left(\frac{\partial A}{\partial q_i}\frac{\partial C}{\partial p_i} - \frac{\partial A}{\partial p_i}\frac{\partial C}{\partial q_i} \right)$$

$$\{A, C\} = -\{C, A\} \quad \{A, A\} = 0 \quad \{kA, C\} = k\{A, C\}$$

$$\{(A+B), C\} = \{A, C\} + \{B, C\}$$

$$\{AB, C\} = B\{A, C\} + A\{B, C\}$$

$$\{q_i, p_j\} = \delta_{ij}$$

$$\{q^n, p\} = n q^{(n-1)}$$

also: $\{q_i, q_j\} = 0$, $\{p_i, p_j\} = 0$

δ_{ij} - kronecker delta function $\delta_{ij} = 0$ if $i \neq j$ $\delta_{ij} = 1$ if $i = j$

possion bracket of quantity with anaother one which is invariant in some translation gives you the derivative of first quantity with respect to the property in which second shows translational invariance   eg.) H - time translational invariance

$\frac{d}{dt} f = \dot{f} = \{f, H\}$

if G conserved in time translations $\{G, H\} = 0$

or $\{x, L_z\} \propto \delta x$

$\{y, L_z\} \propto \delta y$

$\{z, L_z\} \propto \delta z$

Levi-Civita symbol ϵ_{ijk}

if $i=j=k$ $\epsilon_{ijk}=0$

if order clockwise eg $(i,j,k\,/\,j,k,i\,/\,k,i,j)$
$\epsilon_{ijk}=1$

if order anti clockwise eg $(i,k,j\,/\,k,j,i\,/\,j,i,k)$
$\epsilon_{ijk}=-1$

Del ∇ $\quad \nabla_x \equiv \frac{\partial}{\partial x}$ $\quad \nabla_y \equiv \frac{\partial}{\partial y}$ $\quad \nabla_z \equiv \frac{\partial}{\partial z}$

↳ gradient of scalar A is a vector $\vec{\nabla} A = \frac{\partial A}{\partial x}, \frac{\partial A}{\partial y}, \frac{\partial A}{\partial z}$

↳ divergence of vector $\vec{A}$ is a scalar $\vec{\nabla}\cdot\vec{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$

↳ curl of vector $\vec{A}$ is a vector $\vec{\nabla}\times\vec{A}$

$x \to y \to z \to x$

$$(\vec{\nabla}\times\vec{A})_x = \frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}$$
$$(\vec{\nabla}\times\vec{A})_y = \frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x}$$
$$(\vec{\nabla}\times\vec{A})_z = \frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}$$

curl has no divergence so $\vec{\nabla}\cdot(\vec{\nabla}\times\vec{A}) = 0$

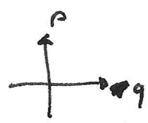
- magnetic field - 0 divergence so represented by vector potential
$\vec{B} = \vec{\nabla}\times\vec{A}$ $\quad B_x = \frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}$
$B_y \ldots$

gauge transformation chages $\vec{A}$ but $\vec{B}$ not affected so $\vec{B}$ gauge tranformation invariant
transformation - gradient function of scalar $\vec{A}' = \vec{A} + \vec{\nabla}\alpha$
↳ transformation scalar

electric field $\vec{F} = -e\vec{\nabla} V$ $\quad$ magnetic field (lorenz force) $\vec{F} = \frac{e}{c}\vec{v}\times\vec{B}$

- gravity $\vec{F}_{grav} = F_{grav}\hat{r}$ $\quad F_{grav} = -\frac{G m_1 m_2}{r^2}$ $\quad \hat{r} = \frac{\vec{r}}{|r|}$ ← or just r (length of radius)
$= -\frac{G m_1 m_2}{r^2}\hat{r}$

- phase space - state space of mechanical system with each point having unique position value and momentum value
  - 2N dimensional



(q,p)

- configuration space - in classical mechanics it is the vector space defined by generalised coordinates describing state of system
  - eucledian or polar
  - N dimentional
- state space - set of all possible configurations (states) of a system
  - variables on axis are state variables

- Hessian matrix $H = \begin{pmatrix} \frac{\partial^2 A}{\partial x^2} & \frac{\partial^2 A}{\partial x \partial y} \\ \frac{\partial^2 A}{\partial y \partial x} & \frac{\partial^2 A}{\partial y^2} \end{pmatrix}$

$$\frac{\partial^2 A}{\partial x \partial y} = \frac{\partial^2 A}{\partial y \partial x}$$

$$\operatorname{Det} H = \frac{\partial^2 A}{\partial x^2}\frac{\partial^2 A}{\partial y^2} - \frac{\partial^2 A}{\partial y \partial x}\frac{\partial^2 A}{\partial x \partial y} = \frac{\partial^2 A}{\partial x^2}\frac{\partial^2 A}{\partial y^2} - 2\frac{\partial^2 A}{\partial x \partial y}$$

$$\operatorname{Tr} H = \frac{\partial^2 A}{\partial x^2} + \frac{\partial^2 A}{\partial y^2}$$

local minimum if : Det H +ve Tr H +ve $\therefore$ min

local maximum if : Det H +ve Tr H -ve $\therefore$ max

saddle point if : Det H -ve $\therefore$ point of inflexion