Phase Shift Method for Measuring Small Capacitance

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***Abstract*—The phase shift method for measuring small capacitances while accounting for the internal capacitance of the oscilloscope is described. An equation for calculating the capacitance of a capacitor from measurable quantities is derived. The experimental method is outlined and the obtained results are used to calculate the capacitance of a small ceramic capacitor to be $C_1 = 551.5 \pm 8.0$ pF. The sources, propagation and effects of uncertainties are discussed. The method is shown to be accurate even for capacitances in the order of 10^{-10} F. Finally a more complicated model of the experimental setup is considered, discussed and shown to reduce to the one initially used.**

I. Introduction

Capacitance is the ability of an electric component to store energy in the form of an electric field caused by displacing charge. Capacitance is usually measured using discharge decay where the voltage across the capacitor is measured over time while the capacitor is discharged through a resistor. The oscilloscope has an internal resistance $R2$ and capacitance $C2$ due to its internal circuitry as shown in Fig. 1. To measure the voltage across the capacitor $C1$ the oscilloscope must be connected in parallel to the capacitor. As a result the measured capacitance and capacitance of the oscilloscope are combined. When the measured capacitance is smaller or equal to that of the oscilloscope this measuring technique becomes inaccurate. An alternative method of measuring small capacitance involves measuring the phase shift between the voltage of the driving signal applied and the voltage across the measured capacitor $C1$. This method accounts for the internal capacitance of the oscilloscope and therefore yields accurate results for significantly smaller capacitances than the decay method.

II. Theory

Capacitive reactance X_c represents the extent to which the capacitance of a component opposes the flow of alternating current. It is related to the capacitance C and the angular frequency of the signal ω as shown in (1) [1]. Equation (2) is obtained by rearranging (1) and writing it in terms of frequency f, by using the relationship $\omega = 2\pi f$. For the experimental setup shown in Fig. 1, X_c is taken to represent the total capacitive reactance of the circuit.C_T is thus the total capacitance of the circuit, given by (3) where C_1 and C_2 are the capacitances of capacitors $C1$ and $C2$ respectively.

The equation for the capacitance of capacitor $C1$ can therefore be written as (4).

$$X_c = \frac{1}{C\omega} \tag{1}$$

$$C = \frac{1}{2\pi X_c f} \tag{2}$$

$$C_T = C_1 + C_2 \tag{3}$$

$$C_1 = \frac{1}{2\pi X_c f} - C_2 \tag{4}$$

The total reactance of the circuit can also be expressed as (5) [2] in terms of the amplitude of the voltage V_c across the capacitors $C1$ and $C2$, the current I_g provided by the function generator and the loss angle ϕ. The loss angle is given by (6) [2]ude of the generator voltage, V_1 is the amplitude of the voltage across resistor $R1$ and α is the phase difference between the voltage across the generator V_g and the voltage across the capacitors V_c.

$$X_c = \frac{V_c}{I_g \cos\phi} \tag{5}$$

$$\cos\phi = \frac{V_g}{V_1}\sin\alpha \tag{6}$$

Equation (7) is obtained by substituting the relationships given by (5) and (6) into (4). Since the resistor $R1$ is connected to the generator and the rest of the circuit in series, the amplitude of the current supplied by the generator I_g is equal to the current I_1 flowing through the resistor $R1$. The fraction $\frac{I_g}{V_1}$ reduces to just the reciprocal of the resistance R_1. Equation (7) can therefore be written as (8) which expresses the capacitance C_1 in terms of known quantities.

$$C_1 = \frac{I_g}{V_1}\frac{V_g \sin\alpha}{2\pi f V_c} - C_2 \tag{7}$$

$$C_1 = \frac{V_g \sin\alpha}{2\pi f V_c R_1} - C_2 \tag{8}$$

The capacitor C_2 and resistor R_2 are not physical components but rather components introduced to account for the physical properties of the channel input circuitry of the oscilloscope and the digital to analogue converter (DAC). The resistance originates from the resistances of all of the components, the input impedance of the DAC as well as the resistance causing leakage current between the traces on the printed circuit board (PCB) and the grounding plane.The capacitance also originates from the combined capacitance of all of the components and the DAC as well as the parasitic capacitance due to conductive surfaces in close proximity to each other such as between the traces on the PCB and grounding planes or EM shielding which acts as a large capacitor place at ground potential.

III. Experimental Method

The experimental method described below closely follows the method outlined in the laboratory manual [2].

The circuit is setup on a breadboard according to the schematic shown in Fig. 1, using components of values specified in Tab. I. The Rohde & Schwarz RTB2004 oscilloscope is used for both driving the circuit and taking measurements. A BNC T junction adapter is connected to the function generator output and a coaxial patch cable is connected to one of the outputs to feed the signal back into channel 2 of the oscilloscope. The second output of the adapter is connected to the circuit on the breadboard as shown on Fig. 1, using a BNC to banana plug cable. Another BNC to banana plug cable is used to connect channel 1 on the oscilloscope across the capacitor $C1$ to measure V_c.

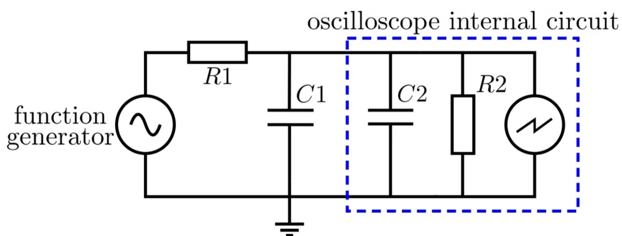

Fig. 1. Diagram showing a schematic representation of the experimental setup.

The function generator is set to produce a sinusoidal wave at a frequency f of 50 kHz. The peak to peak voltage V_{pp} is set to 2 V, with a 0 V voltage offset. Channels 1 and 2 are set to AC coupling. The measurement statistics are set to measure the peak to peak voltage across channel 1 V_{cpp} corresponding to twice the amplitude of V_c, peak to peak voltage across channel 2 V_{gpp} corresponding to twice the amplitude of V_g and to measure the phase difference α between the signal at channels 1 and 2.

The output of the function generator is now switched on and the vertical and horizontal scales are adjusted to fit one full cycle of the signals onto the screen. The measurement statistics are reset and after the mean and standard deviation readings have stabilised a screenshot is captured and downloaded.

IV. Results

The numerical values and their uncertainties for all of the variables required to find C_1 are shown in Tab. I. The screen capture from the oscilloscope together with the data analysis code and photographs of the experimental setup are available at the following GitHub repository [3].

The values of V_g and V_c can be determined by simply halving the corresponding peak to peak values V_{gpp} and V_{cpp} obtained from the oscilloscope screen capture. Since in (8), V_g and V_c appear as a ratio, the factors of 2 cancel out and the peak to peak values may be used directly as shown by (9). The value α is determined directly from the oscilloscope screen capture and only needs to be converted from degrees to radians.

$$C_1 = \frac{V_{gpp}}{V_{cpp}} \frac{\sin \alpha}{2\pi f V_c R_1} - C_2 \tag{9}$$

The value of the frequency f is determined from the settings of the function generator. This value does not include an uncertainty as one was not provided by the manufacturer, the significance of this is discussed later.

The resistance R_1 of the resistor $R1$ through which the capacitor is charged and discharged is determined from the colour code on the resistor.

The value of the capacitance C_2 and the resistance R_2 of the internal circuit of the oscilloscope and their uncertainties are determined from the technical specifications of the oscilloscope [4] [5] provided by the manufacturer. The resistance R_2 is not required to calculate C_1 using (9), however the resistor R2 is still included in Fig. 1 and discussed later, therefore its value is also included in Tab. I.

TABLE I: Numerical Values

Symbol	Value	SI Unit
V_{gpp}	2.0496 ± 0.0054	V
V_{cpp}	1.3174 ± 0.0040	V
α	2.263 ± 0.010	rad
f	5.0×10^4	Hz
R_1	$6.800 \pm 0.068 \times 10^3$	Ω
C_2	$9.0 \pm 2.0 \times 10^{-12}$	F
R_2	$1.00 \pm 0.02 \times 10^6$	Ω

Table containing all numerical values and the associated expected uncertainties.

The final value of the capacitance C_1 along with its associated expected uncertainty is determined by substituting the numerical values in Tab. I into (9) as done in the python code [3]. The final calculated value is $C_1 = 551.5 \pm 8.0$ pF.

V. Uncertainties

The expected uncertainties in the peak to peak voltages V_{gpp} and V_{cpp} are taken to be the standard deviation calculated by the oscilloscope and displayed in the measurement statistics window. These errors are in the order of 10^{-3} V and likely originate from random EM noise induced in the circuit. By observing the blue and red lines in Fig. 2, it can be seen that these uncertainties contribute to the final uncertainty in C_1 significantly however, they are not the major source of uncertainty in this experiment.

Similarly to the peak to peak voltages the expected uncertainty in the phase shift α is taken to be the standard deviation calculated by the oscilloscope. The measurement of the phase shift depends on the relative phases of both signals at channel 1 and 2 therefore there should be minimal systematic error. From the magenta line on Fig. 2, it can be seen this uncertainty has major contributions to the expected uncertainty in C_1.

To decrease the expected uncertainties in the peak to peak voltages as well as phase shift, increase the wave count at which the readings are taken by waiting longer after resetting the measurement statistics. Additionally EM noise may be decreased by using a grounded shielding enclosure however this will lead to an increase in parasitic capacitance.

Resistor $R1$ is assumed to be a standard 0.6W metal film resistor for which the tolerance stated by the manufacturer is $\pm 1\%$. The heating effect due to the current passing through the resistor is negligible therefore the error is random and due to limitations of the manufacturing process. From the cyan line on Fig. 2, it can be seen that uncertainty of R_1 has the greatest contribution to the overall expected uncertainty of C_1. The best way to decrease the uncertainty is to measure the resistance using a precision ohmmeter. Alternatively a precision thin film resistor may be used in place of $R1$. Precision wire wound resistors, although more precise, should be avoided due to large parasitic capacitance and inductance.

The expected uncertainty in the capacitance of the internal circuitry of the oscilloscope C_2 is determined from the technical specifications provided by the manufacturer [4] [5]. The error causing this uncertainty is random originating from the tolerances of the manufacturing process. From the green line on Fig. 2, it can be seen that the expected uncertainty of C_2 has significant contributions to the overall expected uncertainty in C_1. The extent to which it contributes is similar to that of the uncertainties in the peak to peak voltages V_{gpp} and V_{cpp}.

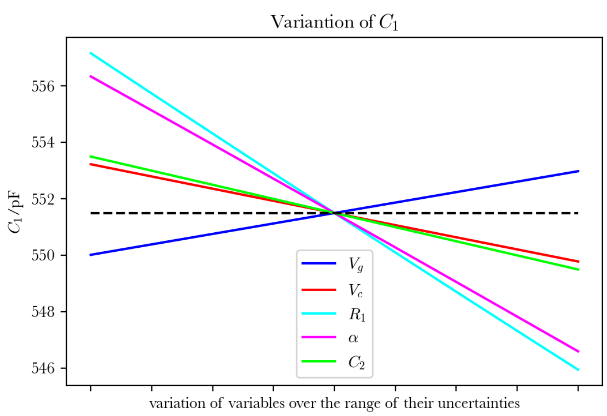

Fig. 2. Plot of the calculated capacitance C_p against variables from (9) varied linearly across the range of their uncertainties.

Both the oscilloscope and signal generator have been tested for zero offset error prior to the measurement. The expected uncertainties discussed above are mostly due to random independent errors. The expected uncertainty in the calculated value of C_1 can therefore be found by using the rule for error propagation in a multivariable function [6]. This calculation is once again done using the python code [3] where the process is outlined in more detail. The partial derivatives for the error propagation are found and computed using the SciPY library.

VI. Discussion

Assuming that the capacitor is not a precision variant, the capacitance stated by the manufacturer Suntan is 470 with a nominal value tolerance of $\pm 20\%$ [7], giving a range of possible values for the capacitance of $376 - 564$.the obtained results of $C_1 = 551.5 \pm 8.0$ pF are therefore within the range of possible values specified by the manufacturer, albeit towards the higher end of the range.

An even more accurate model of the situation can be created by considering the internal impedance of the output stage of the signal generator, represented by a resistor in series with the output, and the internal circuitry of the inputs of both of the oscilloscope channels used as shown on Fig. 3.

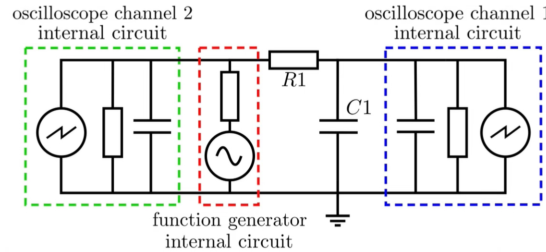

Fig. 3. Diagram showing a more accurate schematic representation of the experimental setup.

The internal resistance of the function generator is 50 Ω [4] and therefore insignificant compared to the 6.8×10^3 Ω resistance of resistor $R1$. If this resistance were accounted the final value of C_1 would be lower.

The internal capacitance of channel 2 oscilloscope input is insignificant since it is connected to the function generator before the resistor $R1$. The current is therefore only limited by the relatively low impedance of the function generator allowing greater current to flow compared to that flowing into channel 1. As a result the capacitance of channel 2 contributes to the overall phase shift less than capacitance of channel 1. If this capacitance were accounted for, the value C_1 would be lower.

Resistances of the oscilloscope inputs are negligible since they are connected in parallel to all capacitors so they do not limit the charging current and since they have a resistance of 1×10^6 Ω their effect on discharging the capacitors is negligible and they may be treated as an open circuit.

An additionally there is parasitic capacitance of the experimental setup which contributing to the overall capacitance measured. Minimising this capacitance will lead to a decrease in the obtained value of C_1

A rigorous mathematical analysis of the factors described above is beyond the scope of this report. Since multiple of the neglected factors lead to an increase in the calculated value of C_1, an experiment should be conducted while minimising these factors and the results should be compared with the ones obtained in this experiment.

VII. Conclusion

A mathematical model for the experimental setup shown in Fig. 1 has been motivated leading to (9) which expresses the capacitance C_1 in terms of measurable quantities. An experiment has been carried out to obtain a set of numerical values from which the capacitance was calculated to be $C_1 = 551.5 \pm 8.0$ pF with a percentage uncertainty corresponding to $\pm 1.45\%$. The low percentage uncertainty as well as the concordance of the obtained results with those specified by the manufacturer show that the phase shift method is capable of measuring small capacitances, in this case of order 10^{-10} F, to an acceptable level of accuracy. In the future the experiment ought to be repeated while implementing the suggested improvements in the uncertainties section and the results compared.

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