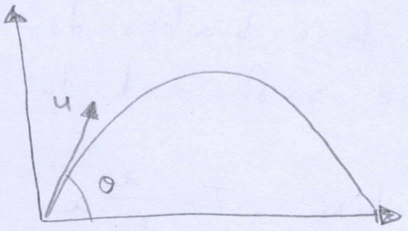
Further M1

- modelling assumptions for projectile motion with acceleration due to g
  - ↳ projectile is pointlike particle
  - ↳ projectile not powered
  - ↳ air has no effect on its motion



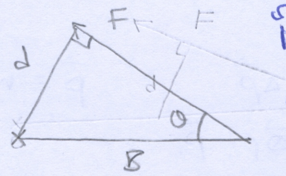
$\vec{v} = \vec{u} + \vec{a}t$ $\qquad$ $\vec{s} = \vec{u}t + \frac{1}{2}\vec{a}t^2$

$\begin{pmatrix} v_x \\ v_y \end{pmatrix} = \begin{pmatrix} u\cos\theta \\ u\sin\theta \end{pmatrix} + \begin{pmatrix} 0 \\ -g \end{pmatrix} t$ $\quad$ $\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} u\cos\theta \\ u\sin\theta \end{pmatrix} t + \frac{1}{2}\begin{pmatrix} 0 \\ -g \end{pmatrix} t^2$ $\quad$ at y_{max} $v_y = 0$ $\frac{x_{max}}{2}$ $\frac{t_{max}}{2}$

x_{max} t_{max} $y = 0$

throught flight V_x = constant

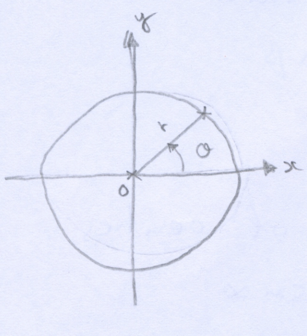
- time to hit ground = 2 × time to reach max height
- equation of path of projectile : $y = x\tan\theta - \frac{gx^2}{2u^2}(1 + \tan^2\theta)$
- $M = Fd = Fs\sin\theta$



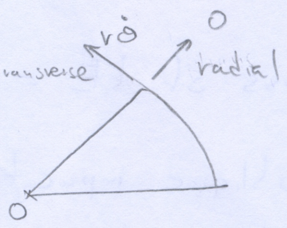
$\overset{\frown}{M}$, $\overset{\frown}{M}$
anticlockwise +ve
clockwise −ve

in equilibrium $\sum M = 0$

- centre of mass - ~~point about which any~~
  - moment of any point about centre of mass is equal to the sum of the moments of particles comprising the body $M\bar{s} = \sum m_i s_i$ $\quad$ $M = \sum m_i$
- in one dimension $M\bar{x} = \sum m_i x_i$ $\quad$ $\bar{x} = \frac{\sum m_i x_i}{M}$ $\quad$ $\bar{x}$ – x coordinate of centre of mass
- in 3D $M\begin{pmatrix} \bar{x} \\ \bar{y} \\ \bar{z} \end{pmatrix} = \sum m_i \begin{pmatrix} x_i \\ y_i \\ z_i \end{pmatrix}$ $\qquad$ $\omega = \dot{\theta}$



$\vec{s} = \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} r\cos\theta \\ r\sin\theta \end{pmatrix}$



$\vec{v}$: transverse component $= r\dot{\theta} = r\omega$
radial component $= 0$

$\vec{a}$: transverse component $= r\ddot{\theta} = r\dot{\omega}$
radial component $= -r\dot{\theta}^2 = -r\omega^2 = -\frac{v^2}{r}$

$v = r\omega$
$\alpha = \omega^2 r = \frac{v^2}{r}$

- by newtons 2nd law of motion: forces are:
  transverse component of force $= mr\ddot{\theta} = mr\dot{\omega}$
  radial component of force $= mr\dot{\theta}^2 = mr\omega^2 = m\frac{v^2}{r}$ (towards centre)

$\mathcal{H} = T + V \qquad T = kE \quad V = PE \qquad \mathcal{H} = \text{constant}$

$= kE + PE$

- circular force brakes don if available force towards cele is $< mr\omega^2$ has to be sufficient to provide centripetal fre as resultant

- Hookes law $\quad T = \frac{\lambda}{L_0} x \qquad \lambda$ - modulus of elasticity

when spring is compressed $x = -ve \qquad T = -ve$ so thrust

$EPE = \frac{1}{2} \frac{\lambda}{L_0} x^2$

- Newtons second law of motion

$$F = ma$$

$$F(t) = m \frac{dv}{dt} \quad \text{- function of time}$$

$$F(s) = mv \frac{dv}{ds} \quad \text{- function of displacet}$$

$$a = \frac{dv}{dt} = v \frac{dv}{ds}$$

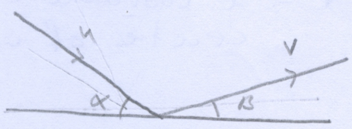
- impulse of a force $= \vec{J} = \vec{F}_{avg}\, t = \Delta \vec{p} = \vec{p}_f - \vec{p}_i \qquad \vec{p} = m\vec{v} \qquad [\vec{p}] = Ns$ or $kgms^{-1}$

- newtons experimental law $\quad e = \frac{v}{u} \quad$ e - coefficient of restitution

$v = eu \quad$ v - speed of separation

u - speed of approach

- collision of sphere - plane



component parallel to surface stays constant:

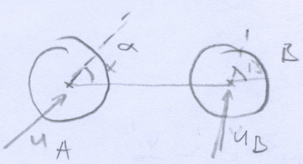
$$u_{\parallel} = u \cos\alpha = v_{\parallel} = v \cos\beta$$

component perpendicular to surface:

$$u_{\perp} = u \sin\alpha \quad v_{\perp} = u_{\perp} e = ue \sin\alpha = v \sin\beta$$

loss of $kE = \frac{1}{2} m u^2 \sin^2\alpha (1 - e^2)$

- oblique impact between 2 spheres



components perpendicular to line of centres remain constant $\quad u_{A\perp} = v_{A\perp} = u_A \sin\alpha$

$u_{B\perp} = v_{B\perp} = u_B \sin\beta$

components along centes of ~~mass~~

momentum conserved $\quad + \quad \vec{v}_B - \vec{v}_A = e(\vec{u}_A - \vec{u}_B)$