Activity Measurement of an Extended ^{90}Sr Source

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***Abstract*—An experiment to determine the activity of an extended ^{90}Sr source is described. A theoretical model of an extended source accounting for absorption by air is fitted onto the data with $\chi^2_\nu = 1.07$ (3sf) to obtain an effective activity of $N = 2.17 \pm 0.09$ MBq. This value is then corrected by accounting for a sheet of Perspex in front of the source giving a final activity of $A = 4.67 \pm 0.11$ Mbq. Random and systematic errors are then discussed as well as future improvements.**

I. Introduction

The errors in most experiments involving radioactive sources will largely depend on the error in the activity of the source. It is therefore desirable to measure the activity with the lowest possible errors. For a non-ideal source this involves complications due to absorption by air and other media in the experimental setup as well as geometric effects caused by the geometry of the source and the detector.

II. Theory

Consider an extended source in vacuum which emits radiation quanta N isotropically. The fraction f of the radiation quanta n detected by a detector is given by

$$f = \frac{n}{N} = \frac{\epsilon\, \Omega}{4\pi} \tag{1}$$

Where Ω is the solid angle subtended by the detector at the position of the source and ϵ the peak efficiency of the detector.

For a flat circular source of radius r_s aligned with a flat circular detector of radius r_d the effective solid angle averaged over the surface of the detector is calculated as [1]

$$\Omega(r) = 4\pi \frac{r_d}{r_s} \int_0^\infty \frac{e^{-rk} J_1(r_d k) J_1(r_s k)}{k} dk \tag{2}$$

Where r is the distance between the center of the source and the detector and J_1 are Bessel functions of 1$^{\text{st}}$ kind of order 1.

Consider a medium between the source and the detector which absorbs some of the emitted particles. The range h of particles of energy E can be calculated using the continuous slowing down approximation (CSDA) as

$$h(E) = \int_0^E \frac{1}{\rho S(E')} dE' \qquad S(E) = -\frac{1}{\rho}\frac{dE}{dx} \tag{3}$$

Where ρ is the density of the medium and S is the mass stopping power as a function of E. For further reading, refer to [2]. The maximum energy E_{max} of particles stopped by the medium can then be found by assuming the thickness of the medium to be the CSDA range h.

III. Experimental Setup

As shown on Fig. 1 the distance d is measured between the edges of the rail carriers using the scale on the optical rail with a random error of ± 1 mm. The distance r is then found as $r = d + \Delta d$ where Δd is a constant offset giving rise to a systematic error in r.

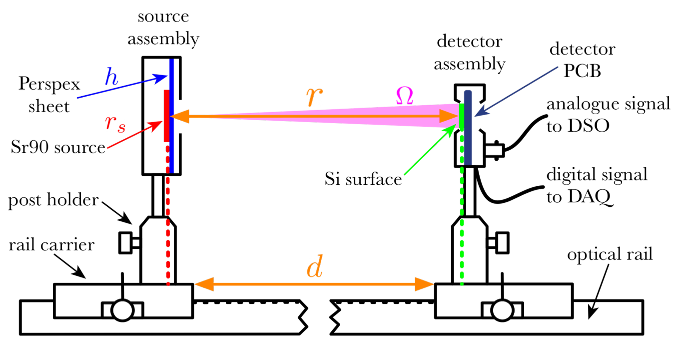

Fig. 1: Side view of the experimental setup with the source and detector mounted on an optical rail used to vary the distance d.

The source assembly consists of an Al backplate which houses the ^{90}Sr behind a sheet of Perspex of thickness $h = 0.97 \pm 0.02$ mm on top of which is an Al front plate with a circular opening of $r_s = 9.17 \pm 0.02$ mm which is taken to be the effective radius of the source. The post is mounted in the center of the backplate resulting in the ^{90}Sr being offset from the edge of the rail carrier by 7.57 ± 0.04 mm.

The detector used is a Hamamatsu S3590-09 PIN Si photodiode [3] with an effective area of 1×10^{-4} m^2. For charged particles the efficiency of PIN photodiode detectors is almost 100%, and since 99.99% of the emitted radiation are electrons, the detector efficiency is taken to be $\epsilon \approx 1$.

To reduce the systematic error in r it is important to determine the offset from the edge of the rail carrier to the center of the photodiode Si surface within the detector assembly. To obtain these measurements a spare detector is dismantled with the help of a laboratory technician. The detector offset is then calculated to be 8.95 ± 0.08 mm giving a total offset of $\Delta d = 16.52 \pm 0.12$ mm.

IV. Methodology

First the source and the detector are brought together and aligned using a ruler to minimise the systematic error from misalignment which would result in the true distance between the source and the detector being greater than measured.

The number of particles detected by the detector n over a measurement time Δt is obtained using a DAQ program in LabView. The measurements are taken for distances d from 0 m up to 0.95 m with increments varying from 5 mm up to 50 mm for a total of 60 measurements.

As the distance d is increased n decreases resulting in an increasing relative uncertainty from the statistical error in n. The measurement time is therefore gradually increased from 10 s up to 180 s in order to ensure at least 2000 counts are measured corresponding to a highest statistical error of 2.24%.

To reduce the systematic error caused by the constant background radiation the source is removed and a background measurement is taken over 300 s giving a background activity of $A_0 = 0.18 \pm 0.02$ Bq.

V. Data Analysis

The data analysis is performed using programs which can be found together with the collected data and other materials in the GitHub Repository [4] for the experiment.

The detector has been measured to have a polaryzable dead time of $\tau = 1.6 \pm 0.3\ \mu$s leading to dead time errors [5] which are corrected for before subtracting the background activity and calculating the adjusted count rate as $\frac{n}{\Delta t} r^2$.

In the point source approximation for $r \gg r_s$ the count rate varies as $\propto \frac{1}{r^2}$ and so in vacuum the adjusted count rate will approach a constant value. When $r \lesssim r_s$ the point source approximation breaks down and the adjusted count rate of an extended source drops off to 0 as $r \to 0$.

The effects of the extended source are therefore accounted for by computing the solid angle Ω from Eq. (2) using numerical integration by truncated trapezium rule. For this the square Si surface of the photodiode is approximated as a disc of equal effective area with radius r_d. An adjusted fraction of radiation detected fr^2 can then be calculated where f is given by Eq.(2). Scaling it up by the effective activity of the source N gives a theoretical adjusted count rate in vacuum as fNr^2.

For a source in air the radiation is absorbed by the air over the increasing distance r resulting in a decrease in the adjusted count rate which can be empirically approximated as linear. Absorption by air can therefore be accounted for by adding another parameter m and writing the theoretical adjusted count rate in air as $f(mr + N)r^2$.

The parameters m and N are determined by fitting the theoretical model onto experimental data using a non-linear least squares method while accounting for the random errors in the measurements. Assuming the random errors follow a Gaussian distribution [6] a reduced chi squared value is calculated for the fitted theoretical model.

To calculate the true activity A of the ^{90}Sr source the fraction of radiation absorbed by the Perspex sheet must be accounted for. This is done by using the ESTAR database [7] to calculate the total mass stopping power of poly(methyl methacrylate) for electrons with energy E in the range 1 keV up to 3 MeV. The analysis program then uses these results to compute the range $h(E)$ from Eq. (3) with relativistic density, using numerical integration. The Newton-Raphson method is then used to calculate the maximum energy E_{max} of electrons stopped by the Perspex sheet in the experimental setup.

The fraction g of electrons absorbed by the Perspex is assumed to be the fraction of electrons emitted with $E < E_{max}$. Energy spectra for ^{90}Sr and ^{90}Y obtained from an online database [8] are combined into a single spectrum which is integrated up to E_{max} giving a value for g.

The true activity A of the ^{90}Sr source behind the Perspex sheet can now be determined by correcting the measured effective activity N as $A = \frac{N}{1-g}$.

The data analysis program propagates all random and systematic errors separately before combining them to determine upper and lower bounds for the calculated activity. The programs for calculations regarding the extended source and the Perspex sheet also include propagation of systematic errors.

Additionally, simulations of the experiment are performed in Geant4 [9] and can be found in the GitHub Repository [4].

VI. Results

From Fig. 2 it can be seen that the effects of systematic errors are greater than those of random errors but their effects on the calculated values are approximately symmetric.

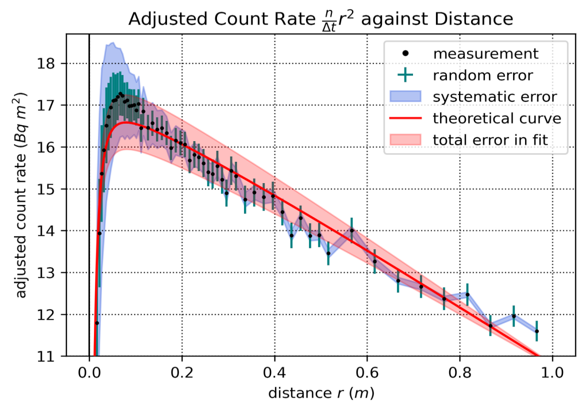

Fig. 2: Adjusted count rate from experimental data and the fitted theoretical curve accounting for geometry of an extended source and absorption by air.

The theoretical curve can be seen to fit the measured data well with a reduced chi squared value of $\chi^2_\nu = 1.07$ (3sf). Minor discrepancies can be seen with the peak of the theoretical curve being lower and wider as well as dropping off faster than the measured data. This is caused by the linear approximation for the effects of absorption by air and could possibly be improved by instead considering a decaying exponential.

There appears to be no horizontal offset between the theoretical curve and the measurements which suggest little systematic error in the alignment and the constant offset Δd.

The effective activity determined by fitting the theoretical model is $N = 2.17 \pm 0.09$ MBq.

For the Perspex sheet in front of the ^{90}Sr it is calculated that the maximum energy of the electrons expected to be absorbed is $E_{max} = 371 \pm 7$ keV. This corresponds to approximately 53.6 ± 0.7 % of the electrons being absorbed and so the effective activity N can be corrected to give a final value for the activity of the source $A = 4.67 \pm 0.11$ MBq.

852 days prior to the experiment the activity of the source has been measured by the ICL safety department to be 5 MBq which has to be considered with care as no errors are provided. Based on this, at the time of the experiment the source is expected to have an activity of approximately 4.72 MBq (3sf) which agrees with the determined value to within the expected uncertainty with a percentage difference of 0.98 % (2sf).

VII. Conclusion

An experimental method to determine the count rate of an extended ^{90}Sr source has been presented. A theoretical model for the adjusted count rate of an extended source in air has been motivated and solved numerically. The model is shown to fit experimental data well with $\chi^2_\nu = 1.07$ (3sf). The effects of absorption by a sheet of Perspex are corrected for, giving an activity for the ^{90}Sr source of $A = 4.67 \pm 0.11$ MBq.

In future experiments the relative statistical error in n can be decreased by increasing the measurement time or using a larger area photodiode detector. The theoretical model can be improved to account for attenuation by air as well as the square shape of the Si surface and effects of field curvature.

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